Report on the Generalized Tanh Method Extended to a Variable-Coefficient Korteweg-de Vries Equation

Bo Tian and Yi-Tian Gao *

Department of Applied Mathematics and Physics, Beijing University of Aeronautics and Astronautics, Beijing 100083, China [a]

* Laboratory of Computational Physics, Institute of Applied Physics and Computational Mathematics, P.O. Box 8009, Beijing 100088, China

Z. Naturforsch. **52a**, 462 (1997); received February 7, 1997

We briefly report that the generalized tanh method can be extended from the situation with coefficient constants to that with coefficient functions. Soliton-typed solutions for a variable-coefficient Korteweg-de Vries equation are thus found. Similar work can be done for the generalized variable-coefficient Kadomtsev-Petviashvili equations.

The variable-coefficient Korteweg-de Vries (vcKdV) equations are able to realistically model various physical situations, as seen, e.g., in [1–4].

A generalized tanh method has newly been proposed and applied to several constant-coefficient nonlinear evolution equations [5, 6]. Hereby, we will extend this method to directly solve for a vcKdV equation ([4] and references therein):

$$u_t = h_1(t)\,(u_{xxx} + 6\,u\,u_x) + 4\,h_2(t)\,u_x - h_0(t)\,(2\,u + x\,u_x), \tag{1}$$

where all $h_j(t)$'s are arbitrary functions. We assume that certain soliton-typed solutions of (1) are of the form

$$u(x,t) = \sum_{j=0}^{N} A_j(t) \cdot \tanh^j [\mathscr{F}(t)\,x + \mathscr{G}(t)], \tag{2}$$

where the $A_j(t)$'s, $\mathscr{F}(t)$ and $\mathscr{G}(t)$ are differentiable functions with $\mathscr{F}(t) \neq 0$ and $A_N(t) \neq 0$, while N is determined via the leading-order analysis as $N = 2$. We then substitute Expression 2 into (1) and equate to zero the coefficients of like powers of x and $\tanh(\mathscr{F}\,x + \mathscr{G})$, so that after computerized symbolic computation we obtain the soliton-typed solutions

$$u(x,t) = \{\beta - 2\alpha^2 + 2\alpha^2 \cdot \operatorname{sech}^2 [\alpha\, x\, e^{-\int h_0(t)\,dt} + 2\alpha(3\beta - 4\alpha^2) \int h_1(t)\, e^{-3\int h_0(t)\,dt}\, dt + 4\alpha \int h_2(t)\, e^{-\int h_0(t)\,dt}\, dt]\} \cdot e^{-2\int h_0(t)\,dt}, \tag{3}$$

where $\alpha \neq 0$ and β are a couple of constants. In [1], the same solutions as (3) were obtained via the inverse scattering. In comparison, the inverse scattering is a well-established, powerful tool, while the technique presented in this note is both concise and straightforward.

Similar work has been done for a generalized variable-coefficient Kadomtsev-Petviashvili equation [7]. We conclude that the generalized tanh method can be successfully extended from the situation with coefficient constants to that with coefficient functions.

Acknowledgements

This work has been supported by the Outstanding Young Faculty Fellowship & the Research Grants for the Scholars Returning from Abroad, State Education Commission of China.

[1] W. Chan and L. Kam-Shun, J. Math. Phys. **30,** 11 (1989).
[2] Z. Chen, B. Guo and L. Xiang, J. Math. Phys. **31,** 2851 (1990).
[3] N. Kudryashov and V. Nikitin, J. Phys. A **27,** L101 (1994).
[4] J. Zhang and P. Han, Chin. Phys. Lett. **11,** 721 (1994).
[5] B. Tian and Y.-T. Gao, Mod. Phys. Lett. A **10,** 2937 (1995).
[6] B. Tian and Y.-T. Gao, Computer Phys. Comm. **95,** 139 (1996).
[7] Y.-T. Gao and B. Tian, to appear in Acta Mechanica (1997).

Reprint requests and correspondence to Prof. Dr. Y.-T. Gao.
[a] Mailing address.

0932-0784 / 97 / 0500-0462 $ 06.00 © – Verlag der Zeitschrift für Naturforschung, D-72027 Tübingen