Nonlocal de Broglie Wavelength of a Two-Photon System

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Abstract

We show that it is possible to associate a de Broglie wavelength to a composite system even when the constituent particles are separated spatially. The nonlocal de Broglie wavelength \( \lambda/2 \) of a two-photon system separated spatially is measured with an appropriate detection system. The two-photon system is prepared in an entangled state in space-momentum variables.

Key words: Quantum Interferometry; Parametric Down-conversion; De Broglie Wavelength; Entanglement.

It is well known that a de Broglie wavelength can be associated not only to single particles, but also to a multiparticle system. For a system of \( N \) identical particles, the resulting wavelength is given by \( \lambda_{DB} = \lambda_i/N \), where \( \lambda_i \) is the de Broglie wavelength associated to the individual constituent particles [1]. Normally, these particles are held together by some kind of binding force as in the experiments done with molecules by Bordé et al. [2] and Chapman et al. [3].

In a recent paper, Fonseca, Monken, and Pádua [4], adapting an original proposal by Jacobson et al. [5], measured the de Broglie wavelength of a two-photon wavepacket for which the role of binding is played by entanglement. A Young interference pattern of two-photon wavepackets, which behaved like single entangled particles, the resulting wavelength is given by \( \lambda_{DB} = \lambda_i/N \), where \( \lambda_i \) is the de Broglie wavelength associated to the individual constituent particles [1]. Normally, these particles are held together by some kind of binding force as in the experiments done with molecules by Bordé et al. [2] and Chapman et al. [3].

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Fig. 1. Double slits arrangements used to produce Young-type interference with pairs of particles. The fourth-order spatial correlation is focused a) on \( x_s + x_i = 0 \); b) on both \( \frac{x_s}{2} + \frac{x_i}{2} = -d \) and \( \frac{x_s}{2} + \frac{x_i}{2} = +d \). For massive particles, this is equivalent to focus their center of mass on a) \( x = 0 \) and b) \( x = \pm d \). The diagonal line shown in b) indicates the case where one of the particles is transmitted and the other is absorbed by the double-slit screen.

In [12] it was shown that the angular spectrum of the pump beam is transferred to the two-photon state generated by SPDC. As a consequence, the probability distribution for two-photon detection \( P_2(x_s, x_i) \) reproduces the transverse pump intensity profile \( W(x) \) in the following way:

\[
P_2(x_s, x_i) \propto W \left( \frac{x_s}{\mu_s} + \frac{x_i}{\mu_i} \right),
\]

where \( \mu_s = \frac{k_p}{k_s} \), \( \mu_i = \frac{k_p}{k_i} \), \( k_p, k_s, k_i \) are the wave numbers of the pump, signal and idler fields, respectively. Regarding the photons as particles travelling with the same velocity, the above expression means that it is possible to control the transverse coordinates of their "center of mass" via the pump beam profile.

Consider that signal and idler photons are incident on two double slits as shown in Figure 1. Let us discuss only the cases in which both photons are transmitted by the slits. By focusing the pump beam on \( x = 0 \), it is possible to force the pair to go either through slits \( A^a_2, A^b_2 \) or \( A^a_1, A^b_1 \) (Fig. 1a). On the other hand, by creating a pump beam profile peaked at both \( x = +d \) and \( x = -d \) (see below), it is possible to force the pair to go either through slits \( A^a_2, A^a_1 \) or \( A^b_2, A^b_1 \) (Fig. 1b). If the two photons are detected on a distant plane, there are, in each case, two indistinguishable paths leading to a coincidence detection. Then, one should expect to see Young-type interference of the photon pair with itself when the detectors are moved. We measured this interference in our experiment.

The experimental setup used is sketched in Figure 2. A 5 mm \( \times \) 5 mm \( \times \) 7 mm BBO nonlinear crystal, pumped by a 200 mW Krypton laser, emitting at 413 nm was used to generate type II SPDC. Down-converted photons with a degenerate wavelength \( \lambda = 826 \) nm propagating at angles of 5° with the pump laser beam direction were selected. Two identical Young double-slits (\( A_1 \) and \( A_s \)) are placed at the exit path of the signal and idler beams at the same distance of 455 mm from the crystal (Figure 2). The
width of each slit and the distance between them are $2a = 0.072$ mm and $2d = 0.26$ mm, respectively. The double-slit planes (approximately in the $xy$ plane) are aligned perpendicular to the plane defined by the pump laser and the down-converted beams ($yz$ plane) with the small dimension of the slits parallel to the $x$-direction (Figure 2). Detectors $D_1$ (idler) and $D_s$ (signal) detect coincidences between the idler and signal photons transmitted through the two double-slits.

Light detectors are avalanche photodiodes, placed at a distance $z_1 = 1060$ mm from the crystal. An arrangement composed of a single collimating slit of width $2b = 0.20$ mm oriented parallel to the Young slits, followed by a microscope objective lens, is placed in front of each detector. $D_1$ and $D_s$ are connected to single and coincidence counters, with a coincidence detection resolving time of 5 ns.

Fourth-order interference patterns were obtained for two different transverse pump beam profiles. The arrangement sketched in Fig. 1a was implemented by focusing the pump beam 460 mm after the crystal. We used a 500 mm focal length lens, placed 40 mm before the crystal. The second arrangement (Fig. 1b) was implemented by projecting the shadow of a wire 460 mm after the crystal so as to create an intensity profile with two peaks, one close to $x = +d$ and the other close to $x = -d$. A 0.125 mm diameter steel wire was aligned parallel to the Young's double-slits and placed 1540 mm before the crystal. A 500 mm focal length lens was placed 540 mm before the crystal (Fig. 2, inset). The pump beam transverse profiles in the $x$-direction, measured at $z = 460$ mm after the crystal can be seen in Figure 3. The profiles were measured by displacing a 0.015 mm diameter pinhole transversely to the beam and measuring with a power-meter the transmitted laser intensity as a function of the pinhole position. The gaussian transverse beam profile with a measured $0.068$ mm FWHM is presented in Figure 3a. In Fig. 3b we see the two-peaks measured intensity profile created by the shadow of the 0.125 mm diameter wire.

Working in the degenerate case ($k_s = k_l = k$) and in the Fraunhofer regime, the number of coincident photons at positions $x_s$ and $x_i$ for the first case (pump beam focused on $x = 0$) can be approximated by [16]

$$N_c(x_s, x_i) = F(x_s, x_i)\left\{1 + \cos \left[k(x_s - x_i)\frac{2d}{z_1}\right]\right\}, \tag{2}$$

where $F(x_s, x_i)$ is the interference function.
where \( z_1 \) is the distance between the double slits and the detectors. In the second case (pump beam with peaks in \( x = +d \) and \( x = -d \)), \( N_c \) can be approximated by [16]

\[
N_c(x_s, x_i) = G(x_s, x_i) \left( 1 + \cos \left( \frac{k(x_s + x_i) 2d}{z_1} \right) \right),
\]

where \( F(x_s, x_i) \) and \( G(x_s, x_i) \) contain diffraction terms. Both expressions (2) and (3) show that we can obtain fringes with a periodicity corresponding to a two-photon de Broglie wavelength by scanning simultaneously both signal and idler detectors. Our results are presented in Fig. 3 (c and d) and Figure 1. In Figs. 3c and 3d, we observe fourth-order interference patterns scanning the position of detector \( D_i \) while keeping \( D_s \) fixed at positions \( x_s = 0.0 \text{ mm} \) (3c) and \( x_s = 1.0 \text{ mm} \) (3d), for the gaussian pump profile of Figure 3a. A similar conditional interference pattern is obtained for the pump profile of Fig. 3b [16].

If the transverse pump profile is the one shown in Fig. 3a, a fourth-order interference pattern with doubled periodicity is obtained by scanning simultaneously the idler and signal detectors in opposite directions with the same step (+\( x_i \) and -\( x_s \), respectively). This is shown in Figure 4a. However, when we move \( D_i \) and \( D_s \) in the same direction (+\( x_i \) and +\( x_s \) directions) with the same step, no interference pattern is observed (Fig. 4b). For the transverse pump profile shown in Fig. 3b the results are the opposite. No interference pattern (Fig. 4c) occurs when we displace the detectors in opposite directions. The Young interference pattern (Fig. 4d) has a period proportional \( \lambda/2 \) when the two detectors are scanned simultaneously in the same direction. Physically, the control of the fourth-order spatial correlation through the transverse intensity profile of the pump laser [12, 18] makes possible the visualization of the pure two-photon effects shown in Figure 4. Also, by manipulating the detection system, different effects are seen: a fourth-order interference pattern with periodicity proportional to \( \lambda/2 \), as well as a situation where we observe no interference pattern at all.

Solid curves in Fig. 4 are theoretical curves with one normalization parameter [16]. The transverse pump profiles at the double-slit position \( W(x, z_A) \) are obtained from the fits of the experimental data (Figure 3). It is important to point out that the Young’s interference pattern shown in Fig. 4a presents the same characteristics as the one shown in Fig. 4d, although

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**Fig. 4.** For the pump beam profile shown in Fig. 3a, fourth-order interference fringes are shown in (a) and (b); for the pump profile of Fig. 3b, they are shown in (c) and (d). (a) and (b) show the coincidence counts as a function of the simultaneous displacement of the detectors in opposite directions (+\( x_i \) and -\( x_s \) directions) and in the same direction (+\( x_i \) and +\( x_s \) directions), respectively. Coincidence counts detection time was 200 s. (c) and (d) show the coincidence counts as a function of the simultaneous displacement of the detectors in opposite directions (+\( x_i \) and -\( x_s \) directions) and in the same direction (+\( x_i \) and +\( x_s \) directions), respectively. Coincidence counts detection time was 1000 s.
they were obtained with different detection procedures. The same occurs in Figs. 4b and 4c. The interference pattern in Fig. 4b has a very small visibility, but not zero. This is due to the finite FWHM of the transverse pump profile (Fig. 3a). Calculation shows that, by narrowing the pump profile even further, the down-converted photons’ correlation is maximized [17, 21]. The dashed curve shows the expected result when the transverse pump profile is a spatial delta function.

Our results can be understood in terms of the physical picture described in Figure 1. For the pump profile of Fig. 3a, the spatial correlation of the generated photon pairs corresponds to the situation depicted in Figure 1a. The interfering pathways described in Fig. 1b correspond to the pump profile of Figure 3b. We have checked this experimentally by detecting the transverse profiles of the twin photons in coincidence at the positions of the double-slits [21]. We can also understand the results of Figs. 4b and 4c. In order to properly define a nonlocal de Broglie wavelength, one minimal requirement is that lengths be defined the same way at each “measuring apparatus”. In the measurements where we see no interference, lengths are defined in opposite directions. For a local system this would be analogous to displacing the detector and then bringing it back to its original position with no net displacement. If one does not maintain the same length definition for observers located at detectors and , any fringe periodicity can be, in principle, observed [22].

In [4], photon pairs were generated collinearly and the fourth-order interference pattern was recorded by displacing the entire “two-photon detector” transversely to the double-slit plane [4, 15]. The “two-photon detector” collects only those photon pairs that fall in the same spatial region defined by its entrance slit. We regard it as a “local detection system”. In the present experiment we measure the de Broglie Wavelength of the biphoton with a “nonlocal detection system”, since the photons belonging to the same pair fall on different spatial regions defined by the slits of each detector ( and ). In [4], the entangled photon pairs interfere like a local single entity. In this work the biphoton interferes like a nonlocal single entity. In both cases we measured the de Broglie wavelength of the biphoton: \( \lambda/2 \).

Our experimental results show that it is possible to define and to measure the de Broglie wavelength of a system of macroscopically separated photons when they are generated in an entangled state in momentum-space variables. The same should be true for massive particles. We have discussed this possibility in a recent work [23]. The concept of a nonlocal de Broglie wavelength introduced here is a generalization of the de Broglie wavelength associated to a multiparticle system. It is not necessary for the particles to be physically bound together or even localized: entanglement is a sufficient ingredient.

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[1] A more general definition of the de Broglie wavelength of a multiparticle system is: \( \lambda_{db} = h/\sum_{i=1}^{N} P_i \), where \( P_i \) is the momentum magnitude of each constituent particle (photons or massive particles).