Generation and Detection of Fock-States of the Radiation Field

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In this paper we give a survey of our experiments performed with the micromaser on the generation of Fock states. Three methods can be used for this purpose: the trapping states leading to Fock states in a continuous wave operation, state reduction of a pulsed pumping beam, and finally using a pulsed pumping beam to produce Fock states on demand where trapping states stabilize the photon number.

Key words: Quantum Optics; Cavity Quantum Electrodynamics; Nonclassical States; Fock States; One-Atom-Maser.

I. Introduction

The quantum treatment of the radiation field uses the number of photons in a particular mode to characterize the quantum states. In the ideal case the modes are defined by the boundary conditions of a cavity giving a discrete set of eigen-frequencies. The ground state of the quantum field is represented by the vacuum state consisting of field fluctuations with no residual energy. The states with fixed photon number are usually called Fock or number states. They are used as a basis in which any general radiation field state can be expressed. Fock states thus represent the most basic quantum states and differ maximally from what one would call a classical field. Although Fock states in analogous cases are routinely observed such as e.g. for the vibrational motion of ions in traps [1], Fock states of the radiation field are very fragile and very difficult to produce and maintain. They are perfectly number-squeezed, extreme sub-Poissonian states in which intensity fluctuations vanish completely. In order to generate these states it is necessary that the mode considered has minimal losses and the thermal field, always present at finite temperatures, has to be eliminated to a large extent since it causes photon number fluctuations.

The one-atom maser or micromaser [2] is the ideal system to realize Fock states. In the micromaser highly excited Rydberg atoms interact with a single mode of a superconducting cavity which can have a quality factor as high as \(4 \times 10^{10}\), leading to a photon lifetime in the cavity of 0.3 s. The steady-state field generated in the cavity has already been the object of detailed studies of the sub-Poissonian statistical distribution of the field [3], the quantum dynamics of the atom-field photon exchange represented in the collapse and revivals of the Rabi nutation [4], atomic interference [5], bistability and quantum jumps of the field [6], atom-field and atom-atom entanglement [7]. The cavity is operated at a temperature of 0.2 K leading to a thermal field of about \(5 \times 10^{-2}\) photons per mode.

There have been several experiments published in which the strong coupling between atoms and a single cavity mode is exploited (see e.g. [8]). The setup described here is the only one where maser action can be observed and the maser field investigated. In our setup the threshold for maser action is as small as 1.5 atoms/s. This is a consequence of the high value of the quality factor of the cavity which is three orders of magnitude larger than that of other experiments with Rydberg atoms and cavities [9].

In this paper we present three methods of creating number states in the micromaser. The first is by way of the well known trapping states, which are generated in a cw operation of the pumping beam and lead
to Fock states with high purity. We also present a second method where the field is prepared by state reduction and the purity of the states generated is investigated by a probing atom. It turns out that the two methods of preparation of Fock states lead to a similar result for the purity of the Fock states. The third method pumps the cavity with a pulsed beam using the trapping condition to stabilize the photon number in our cavity. This method produces Fock states on demand.

II. The One-Atom Maser and the Generation of Fock-States using Trapping States

The one-atom maser or micromaser is the experimental realisation of the Jaynes-Cummings model [10], as it allows to study the interaction of a single atom with a single mode of a high $Q$ cavity. The setup used for the experiments is shown in Fig. 1 and has been described in detail in [11]. Briefly, in this experiment a $^3$He-$^4$He dilution refrigerator houses the microwave cavity which is a closed superconducting niobium cavity. A rubidium oven provides two collimated atomic beams: a central one passing directly into the cryostat and a second one directed to an additional excitation region. The second beam was used as a frequency reference. A frequency doubled dye laser ($\lambda = 294$ nm) was used to excite rubidium ($^{85}$Rb) atoms to the $63\ P_{3/2}$ Rydberg state from the $5\ S_{1/2}$ ($F = 3$) ground state.

Velocity selection is provided by angling the excitation laser towards the main atomic beam at about $11^\circ$ to the normal. The dye laser was locked, using an external computer control, to the $5\ S_{1/2}$ ($F = 3$) $\rightarrow 63\ P_{3/2}$ transition of the reference atomic beam excited under normal incidence. The reference transition was detuned by Stark shifting the resonance frequency using a stabilized power supply. This enabled the laser to be tuned while remaining locked to an atomic transition. The maser frequency corresponds to the transition between $63\ P_{3/2}$ and $61\ D_{5/2}$. The Rydberg atoms are detected by field ionization in two detectors set at different voltages, so that the upper and lower states of the maser transition can be investigated separately.

The trapping states are a steady-state feature of the maser field peaked in a single photon number; they occur in the micromaser as a direct consequence of field quantisation. At low cavity temperatures the number of blackbody photons in the cavity mode is reduced and trapping states begin to appear [11, 12]. They occur when the atom field coupling constant

Fig. 1. The micromaser setup. For details see [9].

Fig. 2. A theoretical plot, in which the trapping states can be seen as valleys in the $N_{ex}$ direction. As the pump rate is increased, the formation of the trapped states from the vacuum can be seen. The positions of trapping states are indicated by arrows with the respective designation $(n, k)$. 
given by the Rabi frequency $\Omega$, and the interaction time, $t_{\text{int}}$, are chosen such that in a cavity field with $n$ photons each atom undergoes an integer number, $k$, of Rabi cycles. This is summarised by the condition

$$\Omega t_{\text{int}} \sqrt{n + 1} = k \pi. \quad (1)$$

When (1) is fulfilled the cavity photon number is left unchanged after the interaction of an atom and hence the photon number is "trapped". This will occur regardless of the atomic pump rate $N_{\text{ex}}$, where $N_{\text{ex}}$ is the rate of pumping atoms in the excited state per decay time of the cavity. The trapping state is therefore characterised by the photon number $n$ and the number of integer multiples of full Rabi cycles $k$.

The build-up of the cavity field can be seen in Fig. 2, where the emerging atom inversion $I = P_g - P_e$ is plotted against interaction time and pump rate; $P_{\text{g(e)}}$ is the probability of finding a ground (excited) state atom. At low atomic pump rates (low $N_{\text{ex}}$) the maser field cannot build up and the maser exhibits Rabi oscillations due to the interaction with the vacuum field. At the positions of the trapping states, the field increases until it reaches the trapping state condition. This manifests itself as a reduced emission probability and hence as a dip in the atomic inversion. Once in a trapping state the maser will remain there regardless of the pump rate. The trapping states show up therefore as valleys in the $N_{\text{ex}}$ direction. Figure 3 shows the photon number distribution as the pump rate is increased for the special condition of the five photon trapping state. The photon distribution develops from a thermal distribution towards higher photon numbers until the pump rate is high enough for the atomic emission to be stabilized by the trapping state condition. As the pump rate is further increased, and in the limit of a low thermal photon number, the field continues to build up to a single trapped photon number and the steady-state distribution approaches a Fock state.

Owing to blackbody radiation at finite temperatures, there is always a small probability of having a thermal photon enter the mode. The presence of a thermal photon in the cavity disturbs the trapping state condition and an atom can emit a photon. This causes the field to change around the trapping condition.

Note that under readily achievable experimental conditions it is possible for the steady-state field in the cavity to approach a Fock state with a high fidelity. Under the present experimental conditions the main deviation from a pure Fock state results from dissipation of the field in the cavity. If a photon disappears it takes a little while until the next incoming excited atom can be used to replace the lost photon. Therefore smaller photon numbers show up besides the considered Fock state. Figure 4 shows micromaser simula-
of such states. Therefore the inversion is generally given by

$$I(n, t_{\text{int}}) = -c \sum_{n} P_n \cos(2\Omega\sqrt{n + 1}t_{\text{int}}), \quad (3)$$

where $P_n$ is the probability of finding $n$ photons in the mode and $t_{\text{int}}$ the interaction time of the atoms with the cavity field. The factor $c$ considers the reduction of the signal amplitude as a result of dark counts.

The experimental verification of the presence of Fock states in the cavity corresponds to a pump-probe experiment in which a pump atom prepares a quantum state in the cavity and the Rabi phase of the emerging probe atom measures the quantum state. The signature that the quantum state of interest has been prepared is simply the detection of a defined number of ground state atoms. To verify that the correct quantum state has been projected onto the cavity, a probe atom is sent into the cavity with a variable, but well defined interaction time. As the formation of the quantum state is independent of interaction time we need not to change the relative velocity of the pump and probe atoms, thus reducing the complexity of the experiment. In this sense we are performing a reconstruction of a quantum state in the cavity using a similar method to that described by Bardoff et al. [15]. This experiment reveals the maximum amount of information that can be found relating to the cavity photon number. We have recently used this method to demonstrate the existence of Fock states up to $n=2$ in the cavity [16].

When the interaction time corresponding to the trapping state condition is met in this experiment, the formation of the cavity field is identical to that which occurs in the steady-state, hence the probe atom should perform an integer number of Rabi cycles. In fact this was observed experimentally [16], which indicates that the pulsed experiment is actually the formation stage of the steady-state experiment. One would therefore expect that the measured photon number distribution, in the dynamical measurement, would be the same as that predicted for the trapping states. State reduction is simply a method of observation that determines the appropriate moment for a measurement. In this sense the observation of a lower emission probability in the steady state is also a field-state measurement as the dip in the steady-state inversion measurement occurs for practically the same conditions as for the dynamical measurements described here [16].
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**IV. Preparation of Fock States on Demand**

In the following we describe another variant of a dynamical Fock-state preparation with the micromaser [17]. To demonstrate the principle of the source described here, Fig. 5 shows the simulation of a sequence of five arbitrary atom pulses using a Monte Carlo calculation in which the micromaser is operated in the (1, 1) trapping state. In each pulse there is a single emission event, producing a single lower-state atom and leaving a single photon in the cavity. The atom-cavity system is then in the trapping condition; as a consequence the emission probability is reduced to zero and the photon number is stabilized. In steady state operation, the influence of thermal photons and variations in interaction time or cavity tuning complicates this picture, resulting in deviations from Fock states [18]. Pulsed excitation, however, reduces the influence of such effects and the generated Fock states show a high purity.

Figure 6 presents three curves, obtained from the computer simulation, that illustrate the behaviour of the maser under pulsed excitation as a function of interaction time for more ideal (but achievable) experimental parameters. The simulations show the probability of finding: no lower-state atom per pulse \( P^{(0)} \), exactly one lower-state atom per pulse \( P^{(1)} \), and the conditional probability of finding a second lower-state atom in a pulse already containing one \( P^{(>1;1)} \). The latter plot of the conditional probability, \( P^{(>1;1)} \), is relatively insensitive to the absolute values of the atomic detection efficiency and therefore has advantages when comparisons with experimental data are performed [17].

It follows from Fig. 6 that, with an interaction time corresponding to the (1, 1) trapping state, both one photon in the cavity and a single atom in the lower-state are produced with 98% probability. In order to maintain an experimentally verifiable quantity, the simulations presented here relate to the production of lower-state atoms rather than to the Fock state left
in the cavity. However, pulse lengths are rather short
\(0.01\tau_{\text{cav}} \leq \tau_{\text{pulse}} \leq 0.1\tau_{\text{cav}}\), so there is little dissi­
pation and the probability of finding a one photon
state in the cavity following the pulse is very close to
the probability of finding an atom in the lower-state.
Atomic beam densities must also be chosen with care
in order to avoid short pulses with high atom density
that could violate the one-atom-at-a-time condition.

Note that at no time in this process is a detector
event required to project the field. The field evolves
in time, to the target photon number state, when a suitable
interaction time has been chosen so that the trapping
condition is fulfilled.

It should be noted that for thermal photon numbers
as high as \(n_{\text{th}} = 0.1\) or for \(t_{\text{int}} = 100\) fluctuations of up to 10% (both beyond the current experimental parameters), simulations show that Fock states are still prepared with an 80-90% fidelity. This is considerably better than for steady state trapping states, where highly stable conditions with low thermal photon numbers are required [12, 11, 18].

The present setup of the micromaser was specifically
designed for steady state operation. Neverthe­
less, the current apparatus does permit a comparison
between theory and experiment in a relatively small
parameter range.

A method was used for the comparison with the experiment which is described briefly in the follow­ing. During the interaction, strong coupling between
pumping atoms and the cavity field creates entangle­
ment between internal atomic levels and the cavity
field. Subsequent pumping atoms will therefore also
become entangled both with the field and a pre­
vious pumping atom. The correlations between subse­
quent atomic levels are determined by the dynamics of
the atom-cavity interaction. The connection between
population correlations and the micromaser dynam­
ics has been studied in detail in [7, 19]. For the present proposal we assume that the cavity is initially empty, and two ex­
cited pumping atoms and the cavity field create entangle­
ment [16] of being unconditional and therefore sig­
ificantly faster in preparing a target quantum state.
Previously, state reduction by detection of a prede­
fined number of lower state atoms was used to pre­
pare the state with 95% fidelity. However, this method
has the disadvantage that it is affected by non perfect
detectors. In the current experiment, however, the cav­
ity field is correctly prepared in 83.2% of the pulses
and is independent of any detector efficiencies. Im­
proving the experimental parameters we can expect to
reach conditions for which 98% of the pulses prepare
single photon Fock states and a single atom in the
lower-state.

VI. An Application of Trapping States
the Generation of GHZ States

The following proposal for the creation of states
of the Greenberger-Horne-Zeilinger (GHZ) type [20]
is an application of the vacuum trapping state. For a
review on the generation of atom-atom entangle­
ment in a micromaser see [7]. For the present proposal
we assume that the cavity is initially empty, and two ex­
cited atoms traverse it consecutively. The velocity of
the first atom, and its consequent interaction time,
is such that it emits a photon with the probability
\((\sin \varphi_1)^2 = 51.8\%\), where \(\varphi_1 = 0.744\pi\) is the corre­
sponding Rabi angle \(\phi\) of \((2)\) for \(n = 0\). The second
atom arrives with the velocity dictated by the vacuum
trapping condition; for \(n = 0\) it has \(\phi = \pi\) in \((2)\), so
that \(\phi = \sqrt{2}\pi\) for \(n = 1\). Assuming that the duration
of the whole process is short on the scale set by the
lifetime of the photon, we thus have

\[
|0, e, e\rangle \xrightarrow{\text{first atom}} |0, e, e\rangle e^{i\varphi_1} - i|1, g, e\rangle \sin(\varphi_1)
\]
$|0, e, e\rangle \xrightarrow{\text{atom}} -|0, e, e\rangle \cos(\varphi_1)$

$-|2, g, g\rangle \sin(\varphi_1) \sin(\sqrt{2} \pi)$

$-i|1, g, e\rangle \sin(\varphi_1) \cos(\sqrt{2} \pi)$,

where, for example, $|1, g, e\rangle$ stands for "one photon in the cavity & first atom in the ground state & second atom excited." With the above choice of $\sin(\varphi_1) = 0.720$, we have $\cos(\varphi_1) = \sin(\varphi_1) \sin(\sqrt{2} \pi) = -0.694$, so that the two components with even photon number $(n = 0$ or $n = 2)$ carry equal weight and occur with a joint probability of 96.3%. The small 3.7% admixture of the $n = 1$ component can be removed by measuring the parity of the photon state $|2\rangle$ and conditioning the experiment to even parity. The two atoms and the cavity field are then prepared in the entangled state

$$\Psi_{\text{GHZ}} = \frac{1}{\sqrt{2}} \left(|0, e, e\rangle + |2, g, g\rangle\right),$$

which is a GHZ state of the Mermin kind [22] in all respects. For a detailed discussion of the method described here see [23].

VI. Conclusion

In this paper we gave a survey of the possibilities for generating Fock states in the micromaser. The generation of Fock states on demand has been experimentally confirmed and will be published elsewhere [17]. The possibility to generate Fock states will allow us to perform the reconstruction of a single photon field or other Fock states in a next step.