Multiparticle Quantum Superposition and Stimulated Entanglement by Parity Selective Amplification of Entangled States

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A multiparticle quantum superposition state has been generated by a novel phase-selective parametric amplifier of an entangled two-photon state. This realization is expected to open a new field of investigations on the persistence of the validity of the standard quantum theory for systems of increasing complexity, in a quasi decoherence-free environment. Because of its nonlocal structure the new system is expected to play a relevant role in the modern endeavor on quantum information and in the basic physics of entanglement. – Pacs: 03.65.Bz, 03.67.-a, 42.50.Ar, 89.70.+c

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Since the golden years of quantum mechanics the interference of classically distinguishable quantum states, first introduced by the famous “Schrödinger Cat” apologue [1], has been the object of extensive theoretical studies and recognized as a major conceptual paradigm of physics [2, 3]. In modern times the sciences of quantum information and quantum computation deal precisely with collective processes involving a multiplicity of interfering states, generally mutually “entangled” and rapidly de-phased by decoherence [4]. In many respects the experimental implementation of this intriguing classical-quantum condition represents today an open problem in spite of recent successful studies carried out mostly with atoms [5-7]. A nearly decoherence-free all-optical scheme based on the process of the quantum injected optical parametric amplification (QIOPA) of a single photon in a quantum superposition state, i.e., a qubit, has been proposed [8, 9]. As a relevant step forward in the realization of the quantum injection scheme, the present work reports a novel optical parametric amplifier (OPA) system that transforms any input linear-polarization entangled 2-photon state (ebit) into a quantum superposition of π-entangled, multi-photon states, indeed an optical “Schrödinger Cat” state (S-Cat) [10]. In order to achieve this result, the new system implements an efficient parity-selective device, usually referred to as “nonlocal entangled interferometer” (NEIF) [11]. In the language of electrical engineering, NEIF conveys on different output channels the squeezed-vacuum noise and the signal, viz. the amplified ebit state. This results in the generation of an S-Cat state with a signal-to-noise ratio (S/N) which may be large, virtually infinite.

Consider the experimental arrangement shown in Figure 1. A nonlinear (NL) beta barium borate (BBO) crystal slab with parallel anti-reflection coated faces, cut for Type II phase-matching and 1.5 mm thick, was excited in both left (L-) and right (R-) directions by an UV mode-locked laser beam which was back-reflected by a spherical UV coated rear mirror MUV with curvature radius (crp) = 30 cm. Precisely, the L- (or R-) amplification is the one determined by the UV beam directed towards the left (or right) in the Figure 1. A computer controlled mount allowed micrometric displacements of MUV along the axis Z, parallel to the wavevector (wv) of the UV beam, kUV.

The UV beam was created by second-harmonic generation of an S-Cat state with a signal-to-noise ratio (S/N) which may be large, virtually infinite.
Consider first the L-amplification of the input vacuum state, viz. the spontaneous parametric down conversion (SPDC) process. This was excited by focusing the UV beam on the right plane surface of the NL crystal by a lens with focal length $f_p = 1$ m. Photon couples in $\pi$-entangled states at a $\lambda = 795$ nm were then generated with an entanglement phase $\Phi'$ equal to the intrinsic QIOPA phase $\Psi$ determined by the spatial orientation of the Type II crystal: $\Phi' = \Psi$ [11, 12]. The dynamical role of $\Psi$ will be defined by the theory given below. The state of each L-emitted photon couple could then be generally expressed as $\left|\Phi'\right> = 2^{-\frac{1}{4}}\left[\left|11.00\right> + \exp(i\Phi')\left|00.11\right>\right]$, where the state $\left|n_1n_2n_3n_4\right> \equiv \left|n_1\right>\left|n_2\right>\left|n_3\right>\left|n_4\right>$ expresses the particle occupancies of the Fock states associated with the relevant $k$-modes, $k_j$ ($j = 1, 2$) with horizontal (h) or vertical (v) linear polarizations ($\pi$). It is well known that the two optical modes belonging to each couple $\{1h, 2v\}$ and $\{1v, 2h\}$ are parametrically correlated, either for L- and R-amplifications, respectively by two equal and mutually independent amplifiers OPA$_A$ and OPA$_B$, the ones that implement the overall action of any Type II (OPA) operating in non-degenerate mode configuration [8, 9, 11]. In the experiment each L-emitted photon pair was selected by a couple of pinholes, back-reflected and re-focused onto the left surface of the NL crystal by two equal spherical mirrors $M_j$ ($j = 1, 2$) with reflectivity 100% at $\lambda$ and radius $c_{M_j} = 30$ cm. Both $M_j$ were placed at an adjustable distance $l \approx 30$ cm from the source crystal. Care was taken to precisely overlap in the NL slab the focal regions (with diameter $\phi \approx 70 \mu m$) of the two back reflected beams at $\lambda$ and of the back reflected UV beam at $\lambda_p$. During the two photon back-reflection and before re-injection of the couple into the QIOPA, a $\lambda/4$ plate with angular orientation ($\varphi$) introduced a change of the entanglement phase of $\Phi''$: $\Phi'' \rightarrow \Phi$, i.e. the original state of the photon couple, $\left|\Phi''\right>$, was transformed just before re-injection into:

$$\left|\Phi\right> = 2^{-\frac{1}{4}}\left[\left|11.00\right> + \exp(i\Phi)\left|00.11\right>\right] \quad (1)$$

The labels $j = 1, 2$ of the orthogonal states in the above spin ($\pi$) representation refer to the optical modes $k_j$. Let us assume now the simple conditions adopted in the experiment: $\varphi = 0$ and $\Psi = \Phi' = 0$. As the phase shifting action of the $\lambda/4$ plate for $\varphi = 0$ leads to $\Phi \rightarrow \Phi' + \pi$ and then, in our case to $\Phi = \pi$, the originally L-generated triplet $\left|\Phi''\right>$ with $\Phi' = 0$ was re-injected into the R-amplifier phase-transformed into the odd-parity singlet: $\left|\Phi_s\right> = 2^{-\frac{1}{4}}\left[\left|11.00\right> - \left|00.11\right>\right]$.

As we shall see shortly, this odd parity ebit $\left|\Phi_s\right>$ is finally R-amplified by QIOPA into a quantum superposition of odd-parity, multi-photon pure S-Cat states: $\left|\Phi\right>_\text{OUT} = 2^{-\frac{1}{4}}\left[\left|\Psi_A\right> - \left|\Psi_B\right>\right]$. In order to complete the overall argument and to clarify the physical origin of the $(S/N)$ selectivity of the system, consider now the R-amplification of the input vacuum state $\left|\text{vac}\right> \equiv \left|00.00\right>$ i.e. the transformation of $\left|\text{vac}\right>$ into the output “squeezed vacuum” state [8, 9]. A quantum analysis shows that this state is represented by a thermal distribution where the parity of the entangled Fock states appearing in the sum is, once again, determined by the intrinsic phase $\Psi$ [13]. Since in our experiment we set $\Psi = 0$, the squeezed vacuum state finally consists of a sum of even-parity states: $\left|\Psi_{\text{vac}}\right>_\text{OUT} \approx -C^{-2}\left[\left|00.00\right> + \Gamma\left|11.00\right> + \left|00.11\right>\right] + \Gamma^2\left|11.11\right> + \Gamma^2\left|22.00\right> + \left|00.22\right> + \cdots$, where $C \equiv \cosh g \approx 1$, $\Gamma = \tanh g < 1$ and the parametric “gain” $g$ are dynamical parameters adopted in the OPA analysis below.

In summary, in our experiment the output “signal”, i.e. the R-amplified ebit, consists of a superposition of odd-parity states while the output “noise”, i.e. the squeezed vacuum, consists of a superposition of even-parity states. At last, NEIF provides the selective addressing of different output channels of the entangled states having different symmetries, i.e. here the ones
expressing respectively the signal and the noise. This is the key idea underlying the parity-sensitive, post-selective properties of the system \cite{8}.

Let us now analyze in more details the QIOPA, viz. the R-amplification process \cite{13}. The two independent amplifiers OPA_A and OPA_B implement the overall OPA process induce unitary transformations, respectively, on two couples of time (t) dependent field operators: \( \hat{a}_j(t) \equiv \hat{a}(t)_{1n} \), \( \hat{a}_2(t) \equiv \hat{a}(t)_{2n} \) and \( \hat{b}_j(t) \equiv \hat{a}(t)_{1v} \), \( \hat{b}_2(t) \equiv \hat{a}(t)_{2v} \) for which, at the initial interaction moment \( t = 0 \) and for any \( i \) and \( j \) and \( i, j = 1, 2 \) is: \( \hat{a}_i \equiv \hat{a}_i(0), \hat{b}_i \equiv \hat{b}_i(0) \) the field operators at the initial interaction time \( t = 0 \). The Hamiltonian of the interaction is expressed in the form: \( H_I = \hbar \chi [\hat{A}^\dagger + e^{i\theta} \hat{B}^\dagger] + \text{h.c.} \), where \( \hat{A}^\dagger \equiv \hat{a}_1^\dagger(1)\hat{a}_2^\dagger(1), \hat{B}^\dagger \equiv \hat{b}_1^\dagger(1)\hat{b}_2^\dagger(1), \theta \equiv \chi t \) is a real number expressing the amplification gain, and \( \chi \) the coupling term proportional to the product of the 2nd-order NL susceptibility of the crystal and of the pump field, here assumed “classical” and undepleted by the interaction. The interaction moment \( t \) may be determined in our case by the length \( l \) of the NL crystal. The quantum dynamics of OPA_A and OPA_B is expressed by the mutually commuting, unitary squeeze operators: \( U_A(t) = \exp[g(\hat{A}^\dagger - \hat{A})] \) and \( U_B(t) = \exp[g(e^{i\theta}(\hat{B}^\dagger - e^{-i\theta} \hat{B}))] \) implying the following Bogoliubov transformations for the field operators: \( \hat{a}_i(t) = C\hat{a}_i + S\hat{a}_j^\dagger \); \( \hat{b}_i(t) = C\hat{b}_i + \hat{S}^\dagger \hat{b}_j \) with \( i \neq j \) \cite{8,9}. Here: \( S \equiv \sinh g, \hat{S} \equiv e^{i\theta} S \).

Of the same dynamics holds for the L-amplification, viz. the SPDC process, generally with a different value of the gain: \( g = \eta g', \Gamma \approx \eta \Gamma' \) being the scaling parameter: \( \eta \approx \sqrt{f_p/r_p} \) and assuming that primed and unprimed parameters refer to the processes of L- and R-amplifications, respectively. The adoption of a scaling parameter \( \eta > 1 \) was found to represent a relevant experimental resource as a larger \( \eta \) leads comparatively to: (a) A larger gain \( g \) of the QIOPA, R-amplification. (b) A smaller gain \( g' \) of the SPDC, L-amplification: implying a smaller emission rate of unwanted SPDC double photon couples. With the adopted value \( \eta = 3 \) the ratio of the SPDC rate of unwanted double photon couples was \( 10^{-2} \) smaller than the rate of single couples, the ones that after back-reflection ad phase-transformation are expressed by the input state \(|\Phi\rangle\).

Let us return to the R-amplification process. By the use of the evolution operator \( U_{AB}(t) \equiv U_A(t)U_B(t) \) and of the disentangling theorem, the quantum injection of the input state given by (2) leads to a Schrödinger-Cat form for the output state:

\[
|\Phi\rangle_{\text{OUT}} = U_{AB}(t) |\Phi\rangle = 2^{-1/2} [ |\Psi_A \rangle + e^{i\theta} |\Psi_B \rangle ] \tag{2}
\]

which, in agreement with the original definition \cite{1,2}, is expressed here as the quantum superposition of the following orthonormal multi-particle states:

\[
|\Psi_A \rangle = C^{-2} \sum_{n,m=0}^{\infty} (S^{-2}n - 1)I^{(n+m+1)} |nn.mm\rangle,
\]

\[
|\Psi_B \rangle = C^{-2} \sum_{n,m=0}^{\infty} (S^{-2}m - 1)I^{(n+m+1)} |nn.mm\rangle.
\]

Note that the phase \( \Phi \) of the input state, (1), and then its parity is reproduced into the output multiparticle state and determines the quantum superposition character of the S-Cat. This phase preserving property appears to be a common feature of all parametric amplification / squeezing transformations of entangled quantum states \cite{8,9}.

We may also inspect the superposition status of the S-Cat by investigating the Wigner function of \(|\Phi\rangle_{\text{OUT}}\). We first evaluate the symmetrically ordered characteristic function of the set of complex variables \( (\eta, \eta^*, \xi, \xi^*) \equiv \{\eta, \xi\} \): \( \chi_{\xi} \{\eta, \xi\} = \langle \Phi | D[\eta(t)]D[\xi(t)] |\Phi\rangle \) expressed in terms of the displacement operators \( D[\eta(t)] \equiv \exp[\eta(t)\hat{a}(0)t] - \eta^*(t)\hat{a}(0)t \) and \( D[\xi(t)] \equiv \exp[\xi(t)\hat{b}^\dagger(0)t] - \xi^*(t)\hat{b}^\dagger(0)t \) where: \( \eta(t) \equiv (\eta C - \eta^* S); \xi(t) \equiv (\xi C - \xi^* S) \) \cite{8}. The Wigner function \( W\{\alpha, \beta\} \) of the complex phase-space variables \( (\alpha, \alpha^*, \beta, \beta^*) \equiv \{\alpha, \beta\} \) is the \( 8 \)-dimensional Fourier transform of \( \chi_{\xi} \{\eta, \xi\} \). By a lengthy application of operator algebra and integral calculus we could evaluate analytically in closed form either \( \chi_{\xi} \{\eta, \xi\} \) and \( W\{\alpha, \beta\} \):

\[
W\{\alpha, \beta\} = \overline{W}\{\alpha\} \overline{W}\{\beta\} \left[ 1 + [e^{i\phi} \Delta\{\alpha\} + \Delta\{\beta\}]^2 - (|\gamma_A^+|^2 + |\gamma_A^-|^2 + |\gamma_B^+|^2 + |\gamma_B^-|^2) \right], \tag{3}
\]

where \( \Delta\{\alpha\} \equiv \frac{1}{2}[|\gamma_A^+|^2 - |\gamma_A^-|^2 - i\text{Re}(\gamma_A^+\gamma_A^-)] \) is given in terms of the squeezed variables: \( \gamma_{A\pm} \equiv (\alpha_1 + \alpha_2^*)e^{\mp \theta}; \gamma_{A\pm} \equiv \bar{i}(\alpha_1 - \alpha_2^*)e^{\pm \theta}. \) Analogous expressions involving \( B \) and \( \beta \) are given by the substitutions: \( A \rightarrow B, \alpha \rightarrow \beta. \) The Wigner functions \( \overline{W}\{\alpha\} \equiv \pi^{-2} \exp(-[|\gamma_A^+|^2 + |\gamma_A^-|^2]); \overline{W}\{\beta\} \equiv \pi^{-2} \exp(-[|\gamma_B^+|^2 + |\gamma_B^-|^2]) \).
\[ \pi^{-2} \exp(-[|\gamma_{B+}|^2 + |\gamma_{B-}|^2]) \], definite positive over the 4-dimensional spaces \( \{\alpha\} \) and \( \{\beta\} \), represent the effect of the squeezed-vacuum, i.e. emitted respectively by OPA\(_A\) and OPA\(_B\) in absence of any injection. Inspection of (3) shows that precisely the superposition character implied by the entangled nature of the injected state \( |\Phi\rangle \), (1), determines through the modulus square term the \( \Phi \)-dependent dynamical quantum interference of the devices OPA\(_A\) and OPA\(_B\), the ones that otherwise act as uncoupled and mutually independent, "macroscopic" objects.

Turn now the attention to NEIF, i.e. to the parity-selective interferometric part of our system which operates over the output beams \( k_j \) emerging from the QIOPA amplifier [11]. Note first that within the present work, NEIF reproduces exactly the Bell-state measurement configuration at the Alice’s site of our original quantum state teleportation (QST) experiment [12]. Consider the field emitted by the NL crystal after R-amplification: Figure 1. The two beams associated with modes \( k_j \) \( (j = 1, 2) \) are generally phase shifted \( \Delta_j = (\psi_{jh} - \psi_{jv}) \) by two equal birefringent plates \( \Delta_j \), and the \( \pi \)-polarizations are rotated by two equal Fresnel-Rhomb \( \pi \)-rotators \( R_j(\theta) \) by angles \( \theta_j \) with respect to directions taken at 45° with the horizontal. The beams are then linearly superimposed by a beam splitter (BS) and coupled by two polarizing beam splitters (PBS) to equal EGG SPCM-AQR14 Si-avalanche detectors \( D_{1h}, D_{1v}, D_{2h}, D_{2v} \) which measures the \( (h) \) and \( (v) \) \( \pi \)-polarizations of the output single modes associated with the field \( \tilde{d}_{1h}, \tilde{d}_{1v}, \tilde{d}_{2h}, \tilde{d}_{2v} \). A computer controlled mount allows micrometric displacements of BS along the axis \( X \). Consider the rate of double coincidences: \( (D_{1h}D_{2v}) \equiv |\Phi\rangle \langle \tilde{N}_{1h}\tilde{N}_{2v}|\Phi\rangle = (D_{2h}D_{1v}), \) where \( \tilde{N}_{1h} = \tilde{d}_{1h}^{\dagger}\tilde{d}_{1h} \). By a detailed account of the full set of transformations induced by the overall system on the input state \( |\Phi\rangle \) we get

\[ (D_{1h}D_{2v}) = \frac{1}{4} \left[ 1 + \cos(\Delta - \Phi) \right] \sin^2(\theta_1 + \theta_2) \]

with \( \Delta \equiv (\Delta_1 - \Delta_2) \). We may check that the phase \( \Phi \) of the input state indeed critically determines the value of this quantity, e.g. by setting \( \Delta = 0, (\theta_1 + \theta_2) = \frac{1}{2} \pi \) the rate reaches its maximum value \( (D_{1h}D_{2v}) \simeq \frac{1}{2} \) for any input even-parity state, e.g. a triplet, \( \Phi = 0 \) while it is zero for any input odd-parity state, \( \Phi = \pi \). This is shown by the data given in Fig. 2 as function of the position \( X \) of the BS. There the width of the resonance expresses the coherence time (225 fs) of the detected photons, which is determined in the experiment by the passband (\( \Delta\lambda = 2 \) nm) of the equal gaussian IF filters placed in front of the D’s. Of course the maximum parity-selectivity is realized when the value of \( X \) \( (X \approx 0, \) in Fig. 2) realizes the \( \text{in principle} \) indistinguishability of the Feynman paths affecting the dynamics of each correlated photon couple before detection. The detailed analysis shows that by adoption of the complementary coincidences \( (D_{1h}D_{1v}) = (D_{2h}D_{2v}) \) the phase-selectivity properties given by (4) are inverted, viz. there an input singlet leads to a resonance peak in Fig. 2, a triplet...
to a dip, etc. [12]. Changes of the selectivity properties can be also realized by appropriate settings of $\Delta_i$ and $\theta_i$ according to (4). An important and unexpectedly large 1st-order quantum interference phenomenon was found when the position $Z$ of the mirror $M_{UV}$ was adjusted to realize the time superposition of the back reflected UV pulse wave-packet (wp) with $w\lambda$ and of the back-reflected SPDC generated wp’s with $w\lambda$ A. A sinusoidal interference fringe pattern with periodicity = $\lambda$ and visibility $V$ up to 40% was revealed by the $D$’s within either single detector and multiple coincidence measurements, see Fig. 2 (right). We explain this striking effect as the realization of the in principle indistinguishability, for any detector’s frame, of the two possible directions over which the detected entangled photon couple was originally emitted: the couple could have been R-generated by the back reflected UV pulse or L-generated by SPDC and then back reflected [14].

Apart from its relevance and novelty, because of its first realization with an entangled state, this effect was helpful to determine the value of $Z$ corresponding to the maximum R-amplification: $Z \approx 0$ in both Figs. 2 and 3. The main R-amplification was carried out with $\Delta = 0$, $(\theta_1 + \theta_2) = \frac{1}{2} \pi$ and investigated by a measurement configuration close to the one $(D_{2v}, D_{2h})$ just considered. Precisely, it was found convenient to adopt a related more complex scheme expressed by the coincidence rate $\mathcal{R}(Z) = (D_{2v}, D_{2h})$ XOR $(D_{1h}, D_{1v})$. This one consists of: (a) A triple coincidence involving 2 detectors $D_{2v}$ normal 50/50 BS: Fig. 1 inset. (b) This triple coincidence was taken in anti-coincidence with either $D_{1h}$ or $D_{1v}$. These options are justified as follows: (a) In (4) the amplified contribution $\propto S^2$ had to be discriminated against the dominant first term arising from the input non amplified single photon couples: this is obtained by the BS technique as shown in [15]. (b) The “noise” coincidence rate due to the output squeezed vacuum is found: $(D_{2v}, D_{2h})_{\text{vac}} = \frac{1}{2} S^2 (1 - \cos \Delta) \cos^2(\theta_1 - \theta_2) + S^4$. Since the last term arises from double detections by $D$-couples involving either $D_{1h}$ or $D_{1v}$, its effect was eliminated by the XOR operation. All this leads, for $\Delta = 0$ to a theoretical noise output value $(\mathcal{R})_{\text{vac}} = 0$, viz. virtually to $S/N = \infty$. Of course this condition implies an ideally perfect alignment of the NEIF, i.e. leading to a 100% visibility ($V$) of the patterns shown in Figure 2. In practice the value $V \approx 0.95$ could be attained so far.

The peak of the signal $\mathcal{R}(Z)$ at $Z \approx 0$ reported in Fig. 3 shows the evidence of the QIOPA amplification of the quantum-injected, $\pi$-entangled state $|\Phi\rangle_s$ into the generation of the multipartite, $\pi$-entangled Schrödinger Cat state: $|\Phi\rangle_{\text{OUT}} = 2^{-\frac{1}{2}} (|\Psi_A\rangle - |\Psi_B\rangle)$. Of course the beast is presently small because of the small value of the gain, evaluated on the basis of the properties of the NL crystal: $g \approx 0.22$. By a series of comparative measurements, carried out in presence and absence of the input UV-pump with $w\lambda$ and ebit-injection beams (wλ), it was found that each input quantum injected photon couple QED stimulated an average number of additional couples $N = 0.20$. With our present experimental conditions the system generates odd-parity entangled 4-photon states $|\Phi_2\rangle = 2^{-\frac{1}{2}} (|22.00\rangle - |02.22\rangle) \equiv 2^{-\frac{1}{2}} (|\uparrow\downarrow\rangle_1 |\downarrow\uparrow\rangle_2 - |\downarrow\uparrow\rangle_1 |\uparrow\downarrow\rangle_2)$ at a rate $\approx 3 \times 10^3 \text{sec}^{-1}$. This result can be linearly scaled however, e.g. by adoption of a more efficient NL crystal and of a more powerful UV source. In the near future at least a factor 17 increase of the value of $g$ shall be attained by the adoption within our system of a standard Ti:S$\alpha$ Regenerative Amplifier Coherent REGA9000 operating with $\delta_i = 150$ f sec pulses, a 270 kHz rep-rate and an average UV output power $\approx 0.30$ W: an apparatus already installed in our laboratory. In this case the gain parameter will increase up to a value very close to its maximum: $\Gamma \equiv \tanh g \approx 1$, and the average number of photon couples QED stimulated by any single injected ebit will be very large $N \gg 1$, as implied by the explicit expressions of the entangled “macrostates” $|\Psi_A\rangle$ and $|\Psi_B\rangle$ given in (2). In conclusion, we have
reported the successful implementation of an entangled Schrödinger Cat apparatus. If the present results will prove to be scalable as expected, the present realization would open a new era of basic investigations on the persistence of the validity of several crucial laws of quantum mechanics for systems of increasing complexity, in a virtually decoherence-free environment [8, 9]. The first test we are performing with the present system concerns the violation of Bell-type inequalities in the multiparticle regime: indeed new perspective in the fundamental endeavour on quantum nonlocality [16].

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