Theoretical Determination of the Mass of a Roton

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For 56 years, researchers have sought, with varied success, to determine the mass of a roton. However, with the recent emergence of the author’s theory of superfluid 4He, the time may soon be at hand when the roton’s mass is determined unequivocally. This letter reports the first quantitative results of the new theory of helium II. Using data from a neutron scattering experiment, the mass of a roton was calculated. It was found that the roton mass is $2.12 \times 10^{-31}$ kg or approximately 0.32 $m_{\text{He}}$, where $m_{\text{He}}$ is the mass of a helium atom.

Key words: Rotons, Special relativity, Superfluid helium-4.

Albert Einstein’s special theory of relativity is one of the two pillars that support all of modern science. In its simplicity, it is unmatched. In its practicality, remarkable. It is sublime and justifiably one of the greatest achievements of the human mind. But is it all? Is it the last word on relative measurements of natural phenomena? Is the speed of light really the only speed in Nature that defines reality for individual observers?

A look at Einstein’s own paper introducing the special theory [1] reveals that he binds the laws of mechanics to the laws of Maxwell’s electrodynamics with the glue that the speed of light is a constant. Hence, it is light that acts as the tool to measure distances and time. Particles in our universe interact, or communicate, via energy quanta, or photons. These photons travel at only one speed, the highest known. Therefore, before a photon from one particle reaches another, there is and can be no transmission of information between the two.

However, modern science has revealed since Einstein’s day that there are interactions between particles which are not electromagnetic in origin. We now know that, indeed, other forms of energy – quanta as well as particles – can and do mediate interactions between particles. In spite of this, though, the assumption that the kinematics involved is always determined by the speed of light, nothing greater, remains a valid one. This aspect of Nature has been confirmed innumerable times, in just as many different types of experiments. Hence, until proven otherwise, we rest assured that the speed of light is the unifying principle which relates one observer’s reality to another’s, while also intimately and vitally binding together all aspects of natural phenomena.

A moment’s thought, however, should be sufficient for one to realize that something very profound lurks behind the benign guise that photons are able to define the kinematics of everything in the Universe. For it is far from clear why the photon alone, of all forms of quanta, should possess this additional and unique property. In fact, there is no a priori reason why there could not exist other energy quanta, besides photons, that define their own systems of kinematics, provided that the quanta travel at speeds less than $c$ and that their spheres of influence are restricted to characteristic environments. But could this be? Could there really exist another realm of natural phenomena in which the interactions of “particles” are describable by the relativistic equations – equations, in this case, that depend on a speed other than $c$? Our answer is yes.

There seems to be tentative evidence [2, 3] that, in fact, at least one example of such a system exists. The system is superfluid 4He. The “particles” in this sub-universe – for lack of a better word to describe a distinct subsystem of natural phenomena – are called rotons, while the quanta of energy that transmit information and, therefore, mediate interactions between rotons are known as phonons.

Presently, the theory which best describes the phenomena associated with helium II is that which makes use of the characteristics of these phonons and rotons. This model is due to Landau [4, 5]. In its ground, or unexcited, state, helium II is taken to be a liquid exhibiting no viscosity or entropy. According to the model, it is by the creation of single rotons and/or phonons, that remove energy and momentum from...
any object moving through the helium, that the phenomenon of superfluidity gradually breaks down; and it is the actions of an effective gas of either phonons and/or rotons that result in helium II's anomalous heat capacity and other characteristics.

An aspect of paramount importance to this theory of elementary excitations is the assumption of the validity of Galilean Invariance, in particular when applied to the motion of phonons. Why Landau rigidly adhered to Galilean Invariance, in spite of the fact that Einstein had shown it to be rigorously incorrect, we have no idea, other than to posit that maybe since phonons need a medium to propagate whilst photons do not, he figured that Lorentz Invariance did not apply. However, we find this to be highly illogical. (Our reasons for this rather unorthodox stance can be found in another, much more detailed exposition that is currently in preparation.) For the purposes of this letter, suffice it to say that, in our opinion, since the phonon is taken to be analogous to a photon, it should therefore behave, not just be described, analogously. In any case, whatever his reason, we find that the abandonment of such a prejudice allows several enigmatic aspects of Landau's theory to be resolved. In fact, it even provides an elegant way to understand why Landau's model works so well.

Starting with the conjecture that the speed of phonons acts as the defining measure for the kinematics of quasi-particles in helium II, we have developed a radically new theory of the liquid [6]. As just a preliminary and simple, but hopefully nonetheless effective, example of the benefits that arise from ascribing a relativity theory to the particles of helium II, we will present our theory's analogue to Einstein's famous equation:

\[ E = mc^2. \]

Prior to our work, Landau's model ascribed to rotons an energy gap \( \Delta \) and a mass \( \mu \), both of which were enigmatic in the sense that \( \Delta \) was undervivable and \( \mu \) had to be empirically found. In our theory, \( \Delta \) is, in fact, derivable and is related to both the speed of sound and the roton mass.

The specific relation which emerges from our theory is

\[ \Delta = \mu_0 s^2, \tag{1} \]

where \( s \) is the speed of phonons in the liquid. With this equation, it is clear that \( \Delta \) should more aptly be called the "rest mass energy of a roton," with \( \mu_0 \) being the "rest mass."

It must be remarked here that the speed of sound varies with the temperature of helium II. But rather than being a liability, this fact is, instead, a windfall. For our theory predicts that Eq. (1) will remain valid throughout the gamut of temperatures that liquid \(^4\)He is superfluid, from absolute zero to \( T_f \), with \( \mu_0 \) assuming the role of a constant of proportionality. Consequently, one can use the energy-mass relation to determine the mass of a roton, quickly and easily, by merely finding the slope of the linear relationship. We have done just this.

With energy gap and speed of sound data from a well-known and finely done experiment [7, 8], we have calculated the roton's mass at three different temperatures of helium II. The results are summarized in Table 1. The table evinces that the mass of a roton is \( 2.12 \times 10^{-27} \) kg. And in terms of the mass of a helium atom, where \( m_{\text{He}} = 6.64 \times 10^{-27} \) kg, it is found that \( \mu_0 = 0.319 \pm 0.005 \) \( m_{\text{He}} \).

Considering the rather bizarre – some would no doubt say preposterous – basis from which Eq. (1) was derived, the resultant estimation of the roton mass is remarkably close to modern estimates found by empirically fitting data. However, one should remain cautious and not be overly content with the result. It must be remembered that data from only three temperatures of helium were used in the calculation, even though our theory predicts a strict temperature independence of the roton mass. Consequently, it is possible that the deduced values appeared only as coincidences. Indeed, the above estimation for the mass of a roton is nearly twice as large as the value found empirically by the authors of the paper from which the data were taken.

### Table 1. Numerical evidence for the relationship between the energy and mass of a roton.

<table>
<thead>
<tr>
<th>Helium temperature ( T ) in K</th>
<th>Energy gap ( \Delta/k ) in K</th>
<th>Speed of sound ( s ) in m/s</th>
<th>Roton mass ( \mu_0 ) in ( 10^{-27} ) kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.08 ± 0.01</td>
<td>8.64 ± 0.10</td>
<td>237.3</td>
<td>2.12 ± 0.02</td>
</tr>
<tr>
<td>1.60 ± 0.01</td>
<td>8.40 ± 0.10</td>
<td>234.3</td>
<td>2.11 ± 0.03</td>
</tr>
<tr>
<td>1.81 ± 0.01</td>
<td>8.23 ± 0.10</td>
<td>231.5</td>
<td>2.12 ± 0.03</td>
</tr>
</tbody>
</table>

* Calculated using Eq. (1).
Nevertheless, the conspicuously consistent repetition of the same roton mass value, at all three temperatures, must be acknowledged. Therefore, in our opinion, the results seem promising.

In conclusion, many other predictions resulting from our theory remain to be tested. It is hoped that researchers will be sufficiently interested to investigate these and more.