Production of Tauonic Uranium in $e^+ - e^-$ Machines* **

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I discuss the production of tauonic uranium in $e^+ - e^-$ machines. A suggested experimental arrangement is given which allows for future machine improvement. To obtain the energy levels to within 1% accuracy, the Dirac equation is solved using a radial uranium charge distribution. Finally, weak $\tau$ capture $P + \tau \rightarrow N + \nu_\tau$ in uranium is calculated.

1. Introduction

Tauonic uranium ($U_\tau$) is the name given to the exotic atom wherein a $\tau$ lepton has replaced an electron. Detection of $U_\tau$ will allow several important experimental results: (i) Precise determination of the $\tau$ mass, (ii) measurement of the $\tau$ magnetic dipole moment (an indicator of possible $\tau$ internal structure), and (iii) search for anomalous non-QED interactions. In regard to the last, it is shown in [1] that Higgs exchange contributes O(eV) corrections to the $1^S$ orbit for

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and subsequent electromagnetic (EM) capture depend only on the speed. This, coupled with the fact that EM capture begins to occur when $e_i < e_k$-shell (i.e. below 20 MeV $\tau$ energy for uranium), permits formation of $U_\tau$, if the tausons can be slowed down sufficiently. From [1], we estimate that 0.5 MeV birth energies should allow sufficient capture cross section. Since a 0.5 MeV $\tau$ goes $\sim 2$ $\mu$m, the birth vertex must be this close to the uranium. Present $e^+ - e^-$ machines are able to focus the colliding beams to within 1 $\mu$m. Future machines will have the capability of focussing to within 0.06 $\mu$m. Thus, an experimental arrangement, Fig. 1, whereby the colliding beams annihilate in a 1 $\mu$m channel in uranium (smaller diameter for future machines) will allow formation of $U_\tau$ if the beam energies are set at threshold.

$\begin{align*}
\textbf{Fig. 1. } e^+ - e^- \text{ annihilation within 1 } \mu \text{m channel in uranium.}
\end{align*}$

Those $e^+$ and $e^-$ not interacting will travel through unhindered. The cross section for threshold production of 0.5 MeV tausons is 0.161 nb. Since the luminosity of future $\tau$ factories is expected to be $10^{33}$ cm$^2$/s, there should be sufficient counting rates.

2. Physics of Capture

Heavy lepton (HL) capture ($\mu^-, \tau^-$) almost never occurs at thermal energies [2], because so few HLs avoid capture in the moderation process. There is a distribution function for the capture HL orbitals ($N_{HL}, l_{HL}$), depending on the incoming HL speed. These distribution functions have a maximum at $N_{HL} = \sqrt{M_{HL}/M_\tau} = 59$ for $\tau$, and flatten out for high incoming speed. However, experimentally, exact distributions are unknown [3]. We must rely on theoretical estimates, but these have only been computed for the lightest elements H, He and Li [4]. It seems to the author that it may be possible to employ the "average-atom" model, which plasma physicists use in computing the properties of laser plasmas, to estimate HL capture rates in high-Z materials; this will not be pur-
sued here. Since the distribution functions will have a maximum at \( \approx N_{59} \), we are interested in the radiative decay of \( \tau \) in \( U_T \) for \( N_{59} \) to \( N_{58} \). This is 0.26 ps, so those tauons never go through the EM cascade to the 1S tauonic orbital (\( N_1 \)). In Table 1, I give the cascade time to the 1S level. We see that \( N_{40} \) is the highest orbital level allowing EM cascade. How often then will the tauons be caught in the inner orbitals? Lacking the distribution functions for high-Z materials, we can only extrapolate from the results for the light atoms [4]. For moderate to high HL incoming energy, the distribution function is flat, beginning from \( \frac{2}{3} N_{\text{max}} \) to \( N_{\text{max}} \). Since \( N_{40} \) is about \( \frac{2}{3} \) of \( N_{59} \), we expect that this will hold true for tauonic uranium for incoming multi-keV \( \tau \) energies.

Some comments about Table 1 are due. First, since the energy difference between adjacent high tauonic principle quantum numbers is not great, Auger (radiationless) transitions are forbidden by energy conservation except for the “shake-off” electrons in the outermost shells. Thus in contrast to muonic atoms, Auger transition rates are small for tauonic uranium and the cascade time is determined by normal dipole radiation transitions. Secondly, by the time that the \( N_{15} \) and inner orbitals are reached, the radiation time is so small in comparison to the 0.3 ps natural \( \tau \) lifetime, that the finite size of the nucleus plays no role in the cascade time. However, the finite nuclear size is important for the energy levels since for \( N_{15} < 7 \), the orbitals lie within the uranium nucleus and the hydrogenic approximation breaks down completely. Instead, the Dirac equation must be solved exactly numerically. Reference [5] gives the uranium protonic charge density for a radial distribution. Using this density, the energy levels are presented in Table 2, while I give in Fig. 2, the Dirac wavefunctions and the uranium charge distribution (\( r_0 = 6.817 \text{ fm} \)).

These are zeroth-order energies, with absolute accuracy to within 1%, but with relative accuracy of much better precision\(^3\). All the higher-order QED corrections apply to \( U_T \), including the use of a more accurate deformed nuclear charge distribution [6]. In this paper, we will not be concerned with them. Looking at Table 2, we observe the spin–orbit splitting, which will give the experimental \( \tau \) magnetic moment. Due to the large value of the \( \tau \) mass, these splittings are small. Since the momentum transfer in the cas-

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\(^3\) The binding energy in the 1S level leads to a \( \approx 7\% \) increase of the \( \tau \) lifetime.
cades is appreciable, nuclear excitations will occur and there will be additional level splittings due to the hyperfine interaction.

Finally, I discuss weak capture \cite{7} $P + \tau \rightarrow N + \nu_{\tau}$ in U$_{\tau}$. This is an important check on e, $\mu$, $\tau$ universality in the weak interactions. The expression for the decay width is given in (5) of \cite{7}, with a characteristic momentum transfer of $q^2 = -0.950 \text{ GeV}^2$. The average protonic density in uranium of the 1S orbital is $q = 19.801/r_0^3$ where $r_0 = 6.8171 \text{ fm}$. We find

$$\Gamma_U = 1.58 \times 10^{10} \text{ s}^{-1}, \quad (1)$$

which should be compared to a value $\Gamma_H = 8 \times 10^9 \text{ s}^{-1}$ for hydrogen \cite{5}. This weak capture in uranium will be accompanied by prompt fission, which is an unmistakable signature.

In conclusion, the formation of tauonic uranium in $\tau$ factories is technologically feasible. It will lead to precise determination of the $\tau$ mass and a check for any anomalous magnetic and electric dipole moments.

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\footnote{This gives an effective $Z_{\text{eff}} = 5.144$.}

\footnote{Weak capture in a nucleus must be faster than hydrogen because the $\tau$ orbit is much smaller, Fermi motion notwithstanding.}