1. Introduction

It is especially appropriate to be discussing the subject of CP violation at this Workshop, since the framework for this part of physics was established more than thirty years ago in a brilliant period of both theoretical and experimental work to which George Sudarshan contributed in a fundamental way. During that period, the central issue of the symmetries that Nature exhibits in the fundamental interactions, and how they may be broken, was recognized. The violation of charge conjugation (C) and parity (P) invariance in the weak interactions was established, and the underlying V−A character of the weak current of fundamental fermions hypothesized and verified. While C and P were separately not conserved, for a relatively brief time it was thought that their combination, CP, might be a symmetry of all interactions. However, in 1964 the experiment [1] of Christenson, Cronin, Fitch, and Turlay showed that CP was violated in decays of the long-lived K meson.

This provided part of the foundation for what we call the Standard Model, wherein the fundamental fermions, quarks and leptons, interact through the exchange of gauge bosons that characterize the weak, electromagnetic, and strong interactions. In the specific case of the unified electromagnetic and weak interactions, the gauge group is \( SU(2) \times U(1) \). Left-handed quarks are doublets of \( SU(2) \), while right-handed quarks are singlets. Consequently, only left-handed quarks partake of the charged-current weak interactions, corresponding to the V−A space-time structure noted above. The weak interactions of the six quarks proceed by emission of a \( W^\pm \) boson that takes the upper (lower) to the lower (upper) component of left-handed, weak-isospin doublets. These are arranged into three generations (or families),

\[
\begin{pmatrix}
  u \\
  d' \\
  s' \\
  b'
\end{pmatrix} =
\begin{pmatrix}
  V_{ud} & V_{us} & V_{ub} \\
  V_{cd} & V_{cs} & V_{cb} \\
  V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
  u \\
  d \\
  s \\
  b
\end{pmatrix},
\]

(1)

where, by convention, the charge +2\( e/3 \) quarks, up (u), charm (c), and top (t) are taken as unmixed, and the weak eigenstates \( d' \), \( s' \), \( b' \), are related to the mass eigenstates of the down (d), strange (s), and bottom (b), quarks with charge \(-e/3\) by the Cabibbo-Kobayashi-Maskawa (CKM) matrix, \( V \), where

\[
\begin{pmatrix}
  d' \\
  s' \\
  b'
\end{pmatrix} =
\begin{pmatrix}
  V_{ud} & V_{us} & V_{ub} \\
  V_{cd} & V_{cs} & V_{cb} \\
  V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix}.
\]

(2)

2. CP Violation in the Three Generation Standard Model

The unitary matrix, \( V \), in (2) that describes the mixing of three generations of quarks can be parameterized in terms of three real angles and one non-trivial phase. Any difference of rates between a given process and its CP conjugate process (or of a CP violating amplitude) always has the form:

\[
\Gamma - \Gamma^c \propto s_1^2 s_2^2 s_3^2 c_1 c_2 c_3 \sin \delta_{\text{KM}}
\]

\[
= s_{12} s_{23} s_{13} c_{12} c_{23} c_{13}^2 \sin \delta_{13}.
\]

(3)

While the quantity on the right-hand side of (3) can be written in a parametrization independent manner [2], we express things explicitly here, first in the original parameterization of the three-generation quark mixing matrix [3], and then in the “preferred”
parametrization adopted by the Particle Data Group [4] using the shorthand that \( s_1 = \sin \theta_1 \) and \( c_1 = \cos \theta_1 \). The quantity \( s_{12} \), or \( s_{12} \), is essentially the sine of the Cabibbo angle [5], \( \sin \theta_c \approx 0.22 \). Our present experimental knowledge [4] assures us that the mixing-angles are small, and the approximation of setting the cosines to unity, which we often adopt in the following, induces errors of a most a few percent. In that case the combination of angle-dependent factors in (3) becomes the approximate combination,

\[
s_{12}^2 s_2 s_3 \sin \delta_{KM} = s_{12} s_{23} s_{13} \sin \delta_{13},
\]

which was recognized [6] earlier as characteristic of CP violating effects in the three-generation Standard Model. Equation (3) shows us that all three generations of quarks are necessary for CP violation; in particular, none of the angles can be zero or ninety degrees, nor can any of the Cabibbo-Kobayashi-Maskawa matrix elements vanish.

The product of Cabibbo-Kobayashi-Maskawa factors in (3) define the “price of CP violation” in the Standard Model. Numerically, it is of order several times \( 10^{-5} \). This “price” might be paid in a specific process by already having many of these angle-factors in both \( \Gamma \) and \( \Gamma' \), corresponding to a very small branching ratio for that process: Then when we form the asymmetry,

\[
A_{CP \; violation} = \frac{\Gamma - \Gamma'}{\Gamma + \Gamma'},
\]

the smallness of the denominator could result in a large asymmetry. On the other hand, the “price” might be paid by having few of these factors in \( \Gamma \) and \( \Gamma' \) separately (and hence in their sum), but only in their difference; the asymmetry will correspondingly be small. There is, therefore, a very rough correspondence between rarer decays and bigger asymmetries. This rule-of-thumb can be mitigated or exacerbated by other factors such as hadronic matrix elements, dependence of one-loop amplitudes upon internal quark masses, and the possible presence of CKM factors that occur in addition to those demanded by (3). A prime example is provided by CP violating effects considered later that depend on \( B - \bar{B} \) mixing, where the large top-quark mass allows big asymmetries between \( B \) and \( \bar{B} \) decays to occur in modes which are not particularly suppressed in rate by CKM factors.

3. The Unitarity Triangle

Unitarity of the CKM matrix, applied to the first and third columns, yields

\[
V_{ub}^* V_{cb} + V_{ud}^* V_{cd} + V_{td}^* V_{tb} = 0 \tag{6}
\]

or, with the \( c_{ij} \) set to unity and thence \( V_{cd} \approx -s_{12} \):

\[
1 \cdot V_{ub}^* - s_{12} \cdot V_{cb}^* + V_{td}^* \cdot 1 \approx 0. \tag{7}
\]

This equation is represented graphically in Fig. 1 in terms of a triangle in the complex plane, the lengths of whose sides are \( |V_{ub}|, |s_{12} V_{cb}|, \) and \( |V_{td}| \). This triangle has been commented upon by many people [7]. It summarizes succinctly the least well-known parts of the CKM matrix. Twice the area of the triangle is

\[
s_{12}^2 s_2 s_3 \sin \delta_{KM} = s_{12} s_{23} s_{13} \sin \delta_{13}.
\]

This is again the “price of CP violation”. If and only if it vanishes, the triangle degenerates to a line and CP is conserved in the three-generation Standard Model.

According to an ancient theorem, perfect measurements of the lengths of all three sides could determine a non-trivial triangle, thereby completely fixing the mixing matrix, including the phase. Alternately, a set of measurements of the lengths could in principle show that the triangle does not exist, forcing us beyond three generations. Unfortunately, given our
The late 1960s and early 1970s marked a peak in experiments on $K$ decays, sparked by the discovery of CP violation [1]. This effort tailed off as many important measurements were completed and new areas of physics opened up in the mid-1970s at electron-positron and hadron machines.

Then in the early 1980s, both theoretical and experimental developments led to a "rebirth" of $K$ physics. On the experimental side, great strides were made to present experimental knowledge together with our limited theoretical ability to compute hadronic matrix elements, the three sides are not known with sufficient accuracy to discriminate between these situations, let alone determine the angles and the area.

With this representation, though, it is possible to see rather directly the interplay and impact of various pieces of experimental information. In particular,

- $|V_{ub}|$ is determined from $b \rightarrow u$ transitions, i.e., $B$ decays to non-charmed states,
- $|V_{cb}|$ is determined from $b \rightarrow c$ transitions, i.e., $B$ decays to charmed states,
- $|V_{td}|$ is determined from $B_d^0 - B_d^0$ mixing,
- $|V_{ts}|$ is determined from $B_s^0 - B_s^0$ mixing. While $|V_{ts}|$ is not a side of the triangle, the small values of the mixing angles imply $|V_{ts}| \approx |V_{cb}|$ to high accuracy.
- $\varepsilon$ – Imposing the constraint that the experimental value of $|\varepsilon|$ characterizing CP violation in the neutral $K$ meson mass matrix has its origin in the Standard Model forcing a non-trivial unitary triangle. However, the precise constraint depends on both hadronic matrix elements and the top quark mass.

A sample of what the presently known experimental data implies for the unitarity triangle is shown [8, 9] in Figure 2. Here, as in Fig. 1b, the base of the triangle has been normalized to unit length (by dividing all sides by $|s_{12} V_{cb}|$, which is about 0.009) and placed along the $x$-axis. This leaves the position of the remaining vertex in the complex plane at $(\varphi, \eta)$ as the quantity we want to determine. Given present experimental and theoretical uncertainties, a considerable domain of shapes and sizes are still allowed for the unitarity triangle.

4. CP Violation in Rare $K$ Decays

The late 1960s and early 1970s marked a peak in experiments on $K$ decays, sparked by the discovery of CP violation [1]. This effort tailed off as many important measurements were completed and new areas of physics opened up in the mid-1970s at electron-positron and hadron machines.

Then in the early 1980s, both theoretical and experimental developments led to a "rebirth" of $K$ physics. On the experimental side, great strides were made to
create high flux beams, handle high data rates, incorporate “smart triggers”, improve detectors (especially for photons), and be able to analyze enormous data samples. These matched, at least to some degree, the requirements in precision and rarity being demanded by the theory for incisive tests of the Standard Model.

On the theoretical side, the establishment of gauge theories for the strong and electroweak interactions provided a well-defined basis for calculations. The three-generation Standard Model could be used to make predictions of what, by definition, was inside, and, by its complement, outside the Standard Model. It was realized that not only did three-generations, provide an origin for CP violation in the nontrivial phase in the quark mixing, but that CP violation should be observable in the $K^0$ decay amplitude [10] as well as the $K^0 - ar{K}^0$ mass matrix. There were also predictions for short-distance contributions to a number of other rare $K$ decay amplitudes induced at one-loop, both CP conserving and CP violating [11].

Given the “price of CP violation”, we can address the question of why CP violation in the $K^0 - ar{K}^0$ mass matrix, characterized by

$$|e| \approx 2.28 \times 10^{-3}$$

is so small, i.e., why CP comes so close to being a good symmetry in the $K$ system. When all the factors are put in, the size of $|e|$ is roughly that of $s_2 s_3 s_6$, which is $\sim 10^{-3}$. This is “naturally” of the right size in the technical sense that, to have $s_2 s_3 s_6$ of order $10^{-3}$ does not require any angle to be fine-tuned to be either especially small or especially large.

This same factor of $s_2 s_3 s_6$ pervades all CP violation observables involving strange particle decays, so it is not so surprising that after more than 25 years the total evidence for CP violation consists of a non-zero value of $e$, and a hint [12] that the parameter $e'/e$ representing CP violation in the $K \rightarrow \pi \pi$ decay amplitude itself is non-zero. Experiments at Fermilab and at CERN are continuing with the aim of reducing the statistical and systematic errors. Theory predicts a non-zero value of $e'$, but the predictions depend not only on the CKM matrix, but on inadequately known hadronic matrix elements and the top quark mass. In other $K$ decays, CP violating effects, although very small, occur with a different weighting (from those in $K^0 \rightarrow \pi \pi$) between effects originating in the mass matrix and decay amplitude. Interesting decays include $K \rightarrow 3\pi$, $K \rightarrow \gamma \gamma$, and $K \rightarrow \pi \pi \gamma$, and especially $K_L \rightarrow \pi^0 \ell^+ \ell^-$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$.

Experimental and theoretical progress over the next few years should clarify the situation for $e'$. But even if the situation becomes true that a non-zero value of $e'$ is established and it is in accord with the three-generation Standard Model, this single number is unlikely to be regarded as conclusively establishing that the origin of CP violation lies in the Cabibbo-Kobayashi-Maskawa matrix. We would demand additional evidence: that a unique set of CKM angles (including the phase) be able to fit several different processes that exhibit CP violating effects, providing a redundant check on the theory.

5. CP Violation in $B$ Decay

The possibilities for observation of CP violation in $B$ decays are much richer than for the neutral $K$ system. In a sense, the situation is reversed, in that for the $B$ system the variety and size of CP violating asymmetries in decay amplitudes for overshadows that in the mass matrix [13].

To start with the more familiar, consider first the phenomenon of CP violation in the mass matrix of the neutral $B$ system. Here, in analogy with the neutral $K$ system, one defines a parameter $e_B$. It is related to $\rho$ and $\eta$, the coefficients of the $B^0$ and $\bar{B}^0$, respectively, in the $B^0$, $\pm q$, $B^0$ eigenstates of the $2 \times 2$ mass matrix, $M - i \Gamma/2$, by

$$q = \frac{1 - e_B}{1 + e_B}.$$  

A meson that is initially a $B^0(\bar{B}^0)$ can, through mixing, acquire a $\bar{B}(B)$ component and decay into a “wrong” sign lepton, $\ell^-$ ($\ell^+$). Correspondingly, a $B\bar{B}$ pair can decay into one “right” and one “wrong” sign lepton, resulting in a pair of same-sign leptons. If, in addition, CP violation is present, the number of $\ell^+\ell^+$ and $\ell^-\ell^-$ pairs can differ and the charge asymmetry in $B^0 \bar{B}^0 \rightarrow \ell^+\ell^- + X$ is given by [14]

$$\frac{\sigma(B^0 \bar{B}^0 \rightarrow \ell^+\ell^- + X) - \sigma(B^0 \bar{B}^0 \rightarrow \ell^-\ell^+ + X)}{\sigma(B^0 \bar{B}^0 \rightarrow \ell^+\ell^+ + X) + \sigma(B^0 \bar{B}^0 \rightarrow \ell^-\ell^- + X)} = \frac{|p/q|^2 - |q/p|^2}{|p/q|^2 + |q/p|^2} = \frac{\text{Im}(\Gamma_{12}/M_{12})}{1 + \frac{1}{2}|\Gamma_{12}/M_{12}|^2} \approx 4 \text{Re} e_B,$$

where we define $\langle B^0 | H | \bar{B}^0 \rangle = M_{12} - \frac{i}{2} \Gamma_{12}$. The quantity $|M_{12}|$ is known from $B - \bar{B}$ mixing to be comparable in magnitude to the $B$ total width, while
$\Gamma_{12}$ is much smaller, receiving contributions only from decay channels which are common to both $B^0$ and $\bar{B}^0$, which are CKM suppressed decay modes. This causes the predicted charge asymmetry for dileptons to be in the ballpark of a few times $10^{-3}$, and at most $10^{-2}$. For the foreseeable future, it is inaccessible experimentally.

Now we turn to where the excitement is: CP violation in decay amplitudes [13]. In principle, this can occur whenever there is more than one path, with different CKM factors, to a common final state. For example, let us consider the favorite paradigm: decay of a neutral $B$ into a CP eigenstate, $f$, such as $\psi K^0$ or $D^+ D^-$. Since there is substantial $B^0 - \bar{B}^0$ mixing, one must consider two quantum-mechanical paths over time from an initial $B^0$ meson to the state $f$:

\[
\begin{align*}
B^0 &\rightarrow f, \\
\bar{B}^0 &\rightarrow f.
\end{align*}
\]

The second path differs in phase both because of $B^0 - \bar{B}^0$ mixing and because the decay of a $\bar{B}$ involves the complex conjugate of the CKM factors involved in $B$ decay. The strong interactions, being CP invariant, give the same phase for the two paths. Adding the amplitudes and squaring, we get interference between the two amplitudes and generate non-zero differences between $\Gamma(B^0(t) \rightarrow f)$ and $\Gamma(\bar{B}^0(t) \rightarrow f)$. Specifically,

\[
\begin{align*}
\Gamma(B^0(t) \rightarrow f) &\sim e^{-\Gamma t} \left(1 - \sin[\Delta m t] \text{Im} \left( \frac{p}{q} \right) \right) \quad (11a) \\
\Gamma(\bar{B}^0(t) \rightarrow f) &\sim e^{-\Gamma t} \left(1 + \sin[\Delta m t] \text{Im} \left( \frac{p}{q} \right) \right). \quad (11b)
\end{align*}
\]

Here we have neglected any lifetime difference, (thought to be very small), between the $B^0$ mass matrix eigenstates, $B^0_{1,2}$, and defined $\Delta m = m_1 - m_2$ and $\varphi = \text{Arg}(B \rightarrow f)/\text{Arg}(\bar{B} \rightarrow f)$. We can form the asymmetry

\[
A_{\text{CP violation}} = \sin 2 \varphi \sin[\Delta m t] \quad (14)
\]

with its characteristic time dependence [15] is big!

What are the angles $\Phi$? They turn out to be nothing but the angles $\alpha$, $\beta$, and $\gamma$ at the vertices of the unitarity triangle! In Fig. 1 a, the angles are labeled by examples of the neutral $B$ decays to CP eigenstates whose asymmetries they govern. Note that if we are to form an asymmetry at all, we must know if we start with a $B^0$ or a $\bar{B}$, i.e., we must “tag” the initial $B^0$. This is a key issue experimentally, as “tagging” efficiencies may be low.

There are a number of other ways to get a CP violating asymmetry in $B$ decays through interference between two paths. One possibility is to have spectator and annihilation graphs contribute to the same process [16]. Still another is to have spectator and “penguin” diagrams interfere [17]. These routes to obtaining a CP violating asymmetry have the advantage that they do not require one to know whether one started with a $B$ or $\bar{B}$, i.e., they do not require “tagging”, for these decay modes are “self-tagging” in that the properties of the decay products (through their electric charges or flavors) themselves fix the nature of the parent $B$ or $\bar{B}$. Their disadvantage, which is significant, is that they generally bring poorly known hadronic matrix elements into the interpretation of an asymmetry, and mar the clean association of the asymmetry with specific combinations of CKM angles and the unitarity triangle.

6. Conclusion

It is more than 25 years since the initial discovery of CP violation. We are still faced with the question of the origin of the CP violating effect seen in the neutral $K$ system and its ultimate significance.

- Is it a curiosity? Could it be physics originating at a much higher mass scale, not tied to the Standard Model, at which we are allowed only a peek – a tiny remnant of new physics beyond the Standard Model?

or

- Is it a cornerstone of the Standard Model? Does it reflect the presence of three generations of quarks
and leptons, with all quark masses unequal and all weak mixing angles non-zero? Is it then the single statement summarizing all of this and yielding a characteristic pattern of CP violation that is tied to quark flavor?

These are the questions that we seek to answer. The main thrusts toward obtaining answers are:

- For $K$ decays, pursuit of the next generation of experiments that have the sensitivity to establish a non-zero value of $\rho/\epsilon$, see a CP violating effect in $K_L \to \pi^0 e^+ e^-$, and even harder, in $K_L \to \pi^0 \nu \bar{\nu}$. With several groups proposing to get to the required levels, we are likely to get interesting results over the next several years.

- For $B$ decays, building the required electron-positron collider operating near $B \bar{B}$ threshold – a $B$ Factory. With a luminosity that generates several times $10^7 B \bar{B}$ pairs per year, some of the asymmetries that we discussed should be clearly seen [18]. At hadron machines that exist or are proposed, the requisite numbers of $B$'s are there, but we must learn how to trigger on those events with $B$'s and reconstruct them. Special detectors that are capable of measuring, if not triggering upon, the $B$ decay-vertex are essential.

Most of the CP violating phenomena in the Standard Model remain to be explored. Over the next decade, I believe that we will see many new examples of CP violating effects. In the process, the full CKM matrix will be elucidated, with perhaps, hints of what lies beyond the Standard Model.

[9] We use $|V_{ub}| < 0.16 |V_{ub}|$, $B_u^{1/2} B_u = 150 \pm 50$ MeV, $m_e = 1.4$ GeV and $\gamma < B_x < 1$ for parameters that enter the various constraints.