Particles and Charges in the Vortex Sponge
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The vortex sponge, which postulates a steady state of vortex turbulence in an ideal fluid, is a substratum invented to explicate parsimoniously “action at a distance” and especially vacuum electromagnetics. A disturbance of the vortex sponge’s structural elements is shown to behave like a Schrödinger quantum particle. Vorticity reversal points model elementary electrical charges.

1. Substratum Approach

The substratum method of modeling physical fields and particles rests on the idea of a material medium which fills without gaps all space. This universal continuum is usually referred to as a substratum, aether or physical vacuum. In the substratum approach physical fields are modeled by various kinds of deformation of the substratum, while particles of matter are associated with soliton-like excitations of the substratum.

One of the more promising small-scale mechanical models of the substratum is represented by a steady state of vortex turbulence of an ideal incompressible fluid, historically called the vortex sponge [1]. It has been shown [2] that this model is able to simulate vacuum electromagnetics. A soliton-like excitation of the vortex sponge’s fine structure will be shown below to behave like a Schrödinger quantum particle. Other features of the vortex sponge mimic those of a quantum vacuum of the conventional theory.

2. Methods to Handle Turbulence

Usual methods based on a rigorous hydrodynamical \textit{ab initio} approach fail to provide information about the fine structure of a turbulent state of a liquid. However, they predict the tendency of a turbulence to self-organization [3].

As in any many-body problem, in order to promote a solution one must accept a priori some constitutional assumptions. In the case of turbulence, we may make a conjecture about vorton structure of the fluid. This means viewing the turbulent fluid as a system of eddies treated like quasicorpuscles [4, 5]. At this point there may be calculated the statistics of “spins” or the system may even be described in terms of creation-annihilation operators with the Hamiltonian just as in quantum electrodynamics [5]. A simple statistics of spin corpuscles has been shown [6] to be a tentative model of electrostatic interaction.

The vortex sponge in its proper sense appears as a result of another assumption concerning the coherency of the system of eddies. Separate eddies are supposed not only to interact with each other, as it follows from hydrodynamics [4] but, what is more, they unite in linear chains such as shown in Figure 1. Thus, the vortex sponge may be viewed [2] as a heap of randomly oriented straight chains of eddies, or vortex filaments (Figure 2). It is the dynamics of the latter secondary structure, as contrasted with the dynamics of the primary fluid, that is of interest here.

We consider first the dynamics of a single isolated vortex filament or chain of eddies and then generalize it to three dimensions.

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Fig. 1. A chain of eddies.

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3. Vortex Filament

The motion of a vortex filament is governed by a dependence of the velocity $\mathbf{V}$ of the vortex filament's liquid element on the curve's local form. To express such a law analytically one needs to describe the vortex filament as a space curve in the usual Frenet-Serret frame.

First, a point on a spatial curve is defined by the position vector $\mathbf{r}$, which is a function $\mathbf{r}(l)$ of the length $l$ measured from a fiducial point along the curve. For a moving curve, there is a further dependence $\mathbf{r}(l, t)$ on the time $t$. Excluding information about the curve's space position, the local form of the curve is fully specified by its curvature $k(l, t)$ and torsion $\tau(l, t)$. The latters are defined through the two unit vectors, a tangent $\mathbf{e}(l, t) = \partial \mathbf{r}/\partial l$ and principal normal $\mathbf{n}(l, t)$, by the Frenet-Serret formulae

$$k \mathbf{n} = \partial \mathbf{e}/\partial l, \quad \tau \mathbf{n} = -\partial(\mathbf{e} \times \mathbf{n})/\partial l,$$

$$|\mathbf{e}| = 1, \quad |\mathbf{n}| = 1.$$  \hspace{1cm} (3.1)

The motion of the vortex filament without stretching is described in these terms by

$$\mathbf{V}(l, t) = \frac{\partial \mathbf{r}}{\partial t} = v_1 k \mathbf{e} \times \mathbf{n},$$  \hspace{1cm} (3.2)

where $v_1$ stands for the coefficient of local self-induction and $\mathbf{e}$ is assumed (in absence of inversion) to be parallel to the filament's vorticity vector $\mathbf{e}$ (Fig. 3; for a rigorous derivation see [8]). Differentiating the latter expression with respect to $l$ and using (3.1), we get a positionally invariant form of the motion law

$$\frac{\partial \mathbf{e}}{\partial t} = v_1 \mathbf{e} \times \frac{\partial^2 \mathbf{e}}{\partial l^2}.$$  \hspace{1cm} (3.3)

It is hard to judge from the motion equations (3.3) and (3.1) the character of the vortex filament's spacial and temporal evolution. Fortunately, these equations can be converted to a form well known in nonlinear mechanics. It was shown rigorously [9] that (3.3) and (3.1) are transformed to the nonlinear Schrödinger equation

$$-\frac{i}{v_1} \frac{\partial \psi}{\partial t} = \frac{\partial^2 \psi}{\partial l^2} + \frac{1}{2} |\psi|^2 \psi$$

under the substitution

$$\psi = k \exp \left[ i \left( \int_0^l \tau \, dl - \omega t \right) \right],$$

where $\omega = \text{const}$ is an energy integral of motion.

Equation (3.4) possesses a soliton solution which corresponds to a hump or loop (Fig. 4 b) of the vortex filament. If it has a nonplanar configuration, the disturbance moves along the vortex filament with local velocity

$$v(l, t) = 2 \tau v_1.$$  \hspace{1cm} (3.6)

As in case of the ordinary Schrödinger equation, the wave function $\psi$ keeps its norm in the course of motion: $\int |\psi|^2 \, dl = \text{const}$. 

4. Chain of Eddies

From another point of view, the structural unit of the vortex sponge can be regarded as an ordered chain of eddies of an ideal fluid (Figure 1). Separate point eddies interact with each other like vector corpuscles, reminiscent of infinitesimal elements of an electric current [4]. However, if properly framed, the corre-
Fig. 4. Single-filament models of (b) a neutral particle and (c), (d) electrical charges with correspondence to (a) a distribution of curvature along the vortex filament. Arrows show a direction of vorticity.

Let us consider a chain of coordinated spin corpuscles with a fixed one-dimensional space coordinate \( x \) of a corpuscle but variable orientation \( e(x, t) \) of its spin, regarded as a continuous function of \( x \), so that \( |e| = 1 \). In case of interaction with nearest neighbours only the Hamiltonian per one spin is

\[
\varepsilon + \varepsilon_0 = -\frac{1}{2} Q \left( \frac{v_1}{\Delta x} \right)^2 \left[ e(x - \Delta x, t) e(x, t) + e(x, t) e(x + \Delta x, t) \right],
\]

and the equation of motion is

\[
\frac{\partial e(x, t)}{\partial t} = \frac{v_1}{(\Delta x)^2} \left[ e(x, t) \times e(x - \Delta x, t) + e(x, t) \times e(x + \Delta x, t) \right],
\]

where \( \Delta x \) stands for the distance between adjacent corpuscles, \( Q \) is the linear density of spin corpuscles, and \( v_1 \) is an analog of the Weiss constant in the theory of ferromagnetism.

Expanding \( e(x \pm \Delta x) \) over \( \Delta x \) near \( e(x) \) and pushing \( \Delta x \) to zero yields

\[
\varepsilon = \frac{Q v_1^2}{2} \left( \frac{\partial e}{\partial x} \right)^2,
\]

\[
\frac{\partial e}{\partial t} = v_1 e \times \frac{\partial^2 e}{\partial x^2}.
\]

The relation \( \partial^2 e^2 / \partial x^2 = 0 \) was used in the derivation of (4.3), and a self-interaction term \( \varepsilon_0 \sim e^2 \) was subtracted.

Besides the energy density \( \varepsilon(x, t) \), the corresponding current density can be found from the continuity equation

\[
\frac{\partial \varepsilon}{\partial t} + \frac{\partial j}{\partial x} = 0
\]

and (4.4):

\[
j(x, t) = \rho v_1^2 e \left( \frac{\partial e}{\partial x} \times \frac{\partial^2 e}{\partial x^2} \right).
\]

Thus, the motion equation (4.4) of the continuum spin system turns out to have the same form as (3.3) of the vortex filament. Now, defining the deformation \( k(x, t) = |\partial e / \partial x| \) and the reduced current velocity \( \tau(x, t) = j / (k^2 \rho v_1^2) \), (4.4), (4.3), (4.5) can be subjected to the same transformation (3.4), (3.5) as (3.3), (3.1) [10]. It can be seen that in this model a one-dimensional space coordinate \( x \) is not necessarily a length along the curve but may be rectangular coordinate of a spin in the straight chain of spins (see e.g. Figure 5 b).

Now, returning to the chain of eddies, it should be noted in retrospect that the original Hamiltonian and the motion equation of the point eddies [4] can be represented in the form (4.1) and (4.2) if the space...
coordinate $x$ is treated as a length $l$ along the curved chain (Figure 1). Thus we obtain in another way the vortex filament’s motion equations (3.3) and (3.4).

It is worth noting that a form closely related to (3.4) with $l = x$ was assumed [11] as a basic equation in a substratum regarded as a packing of tiny vortex rings (point vortex dipoles). A vortex dipole model of turbulence is hydrodynamically more correct than that of point eddies (vortons) [12], owing to the vanishing of divcurl of the fluid velocity.

### 5. Self-Energy Interpreted as Mass

According to (3.2) or (4.3), the part of the kinetic energy of the primary fluid due to distortion of the vortex filament is

$$E_0 = \frac{1}{2} \rho \int V^2 \, dl = \frac{1}{2} \rho v_1^2 \int k^2 \, dl$$

$$= \frac{1}{2} \rho v_1^2 \int (\partial e/\partial l)^2 \, dl = \frac{1}{2} \rho v_1^2 \int |\psi|^2 \, dl = \text{const} \, , \quad (5.1)$$

where $\rho$ stands for the linear density of the fluid along the vortex filament. This energy has the meaning of the self-energy of the disturbance and may be interpreted as the mass $m$ of this disturbance. Thus the density

$$\frac{1}{2} \rho V^2(l, t) = \frac{1}{2} \rho v_1^2 k^2(l, t) = \frac{1}{2} \rho v_1^2 (\partial e/\partial l)^2 \quad (5.2)$$

of the distribution of the distortion energy along the vortex filament is identified with the linear density of the space distribution of the soliton mass.

### 6. Secondary Invariants of Motion

After an $m/E_0$ renormalization, another integral of the vortex filament’s motion, $\int k^2 \tau \, dl$, which has no clear interpretation in terms of the density flow

$$\frac{1}{2} \rho V^2_v = \rho v_1^4 k^2 \tau$$

of distortion energy (see (5.2) and (3.6)), is treated as translational momentum

$$\int \frac{1}{2} \rho V^2 \frac{mv \, dl}{E_0} = \frac{2mv_1}{\int k^2 \, dl} \int k^2 \tau \, dl$$

d of the disturbance and serves as a starting point to distinguish between the characteristics of motion of the vortex filament and the characteristics of motion of a soliton disturbance of this filament.

Besides the translatory motion associated with the deviation of the vortex filament from a planar configuration (see (3.6)), the disturbance performs another kind of motion determined by the gradient of the filament’s curvature. Namely, the disturbance spreads about diffusely with the local velocity $u(l, t)$ given by the Fick’s law of diffusion

$$k^2 u = -v_1 \partial k^2 / \partial l \, . \quad (6.1)$$

The latter does not contribute to the disturbance’s total momentum:

$$\int k^2 u \, dl = -v_1 \int_{-\infty}^{+\infty} (\partial k^2 / \partial l) \, dl = 0 \, .$$

However, its contribution $E_d$ to the energy $E$ of the disturbance motion does not vanish:

$$E_d = \int \left[ \frac{1}{2} \rho \frac{V^2}{E_0} \right] \frac{1}{2} m u^2 \, dl = 2mv_1 \int \frac{(\partial k^2 / \partial l)^2 \, dl}{\int k^2 \, dl} \, .$$

The sum $E$ of the soliton’s translational energy

$$E_i = \int \left[ \frac{1}{2} \rho \frac{V^2}{E_0} \right] \frac{1}{2} m v^2 \, dl = 2mv_1^2 \int \frac{k^2 \tau^2 \, dl}{\int k^2 \, dl} \, .$$

and $E_d$ proves to be an energy integral of motion:

$$E = E_i + E_d = 2mv_1 \tau \, . \quad (6.2)$$

### 7. Generalization to Three Dimensions

A point $r$ of the vortex filament is characterized by the orientation of the fluid’s vorticity which (in the absence of inversion) coincides with the unit tangent vector $e(r, t)$ of the curve. Insofar as the vortex sponge is a three-dimensional medium with a microstructure of a system of randomly oriented vortex filaments, any point $r$ is characterized macroscopically by the triad $\{e_1, e_2, e_3\}$ of the unit vectors $e_j(r, t), j = 1, 2, 3$ (Fig-
Thus the vortex sponge, as the chaotic three-dimensional composition of the ordered one-dimensional subsystems, is described properly by the three equations (3.3) or (4.4),

$$\frac{\partial e_j}{\partial t} = v_1 e_j \times \nabla_j^2 e_j, \quad j = 1, 2, 3,$$

or by the set of corresponding nonlinear Schrödinger equations (3.4)

$$- i \frac{\partial \psi_j}{\partial t} = \nabla_j^2 \psi_j + \frac{1}{2} |\psi_j|^2 \psi_j, \quad j = 1, 2, 3,$$

(here there is no summation over $j$). From the latter set the three-dimensional equation

$$i \frac{\partial \psi}{\partial t} = - v_1 \nabla^2 \psi - \frac{\varepsilon}{v_1} \psi$$

may be composed, where $\psi = \psi_1 \psi_2 \psi_3$ and the energy density $\varepsilon$ is

$$\varepsilon = \frac{1}{2} q v_1^2 \sum_{j=1}^{3} |\psi_j|^2 = \frac{1}{2} q v_1^2 \sum_{j=1}^{3} k_j^2 = \frac{1}{2} q v_1^2 \sum_{j=1}^{3} \left( \nabla_j \psi \right)^2.$$

According to (6.1), any disturbance finds itself eventually in a distributed state with $l_0 k \sim l_0/R \ll 1$, where $l_0$ stands for an effective segment of the polymer chain of eddies and $R$ is the span of the disturbance. For such a delocalized disturbance we may neglect in (3.4) or (7.1) the nonlinear term, which is of relative order of magnitude $k^2$:

$$i \frac{\partial \psi}{\partial t} = - v_1 \nabla^2 \psi.$$

For a curve close to a straight line there is no need to distinguish between the length along the filament and a corresponding rectangular coordinate.

The disturbance moves from one vortex filament to another via the hydrodynamic interaction [4] of distant eddies. The form (3.4) of the single-filament motion equation can only be invariant under generalization to three dimensions if the effective segment is greater than the average minimal distance between neighbouring vortex filaments (Figure 2). Thus, the validity of (7.1) or (7.3) means that the disturbance propagates through the vortex sponge mostly along the vortex filaments with a slight spreading to adjacent filaments. The model is similar to the ideal gas model of statistical mechanics.

8. Aether and Matter

Let us consider the soliton-like excitation of the vortex sponge as a particle of matter.

Thus the aether model of matter turns out to be a two-storey construction with the liquid substratum serving as the ground floor. Indeed, it is not the soliton-like excitations of the primary fluid but, so to speak, excitations of the excitations that are perceived by us as particles of matter.

It can be seen that the energy (5.1) of motion of the vortex filament, named above the self-energy $E_0$ of its excitation and interpreted now as the mass $m$ of a particle of matter, is in fact a part of the kinetic energy of the fluid. (It is computed as an addition to the self-energy of the vortex filament at rest [8].) The kinetic energy (6.2) of motion of a filament’s disturbance has no immediate sense of a kinetic energy of the fluid. Thus the equivalence of mass and energy can not be understood literally as an identity of the two kinds of energy, the self-energy (nuclear energy) of matter and the energy of its motion. It means only that the motion of a particle is always accompanied by the parallel change in its self-energy, such as that connected with the deviation of the loop (Fig. 4 b) from the planar configuration.

Following the broad definition of matter just proposed, the vortex sponge, being disturbed, constantly creates particles in some regions while simultaneously destroying them in others. Still, there must be some topological feature distinguishing true particles from various ephemeral “resonances”.

9. Particles and Charges

The fundamental solution of (3.4) or (7.3) is given by a “wave packet” $k(l, t)$ of Gaussian-like or bell-like shape (Figure 4a). Normally it corresponds to a loop of the vortex filament, which is almost planar when translationally at rest [9] (Figure 4b). According to (3.2), such a loop rotates clockwise around the straight line direction of the vorticity. This configuration is just the single-filament model of an electrically neutral particle. The rotation refers to the spin of a neutron and manifests itself macroscopically as a centre of torsion of the elastic substratum.

There turns out to exist another stable configuration of the oriented spatial curve, virtually corresponding to the same smooth solution $k(l, t)$ of (3.4) or
Fig. 6. Two-dimensional sketches: (a) of a neutral particle and (b) of a charged particle.

Fig. 7. A vortex filament model of the elementary electrical charge: a branch point of a directed curve with osculation and inversion of direction.

(7.3). This is a filament with lateral inversion of vorticity entailing a specific cusp of the curve (Figure 4c, d). Although these figures show a sharp break of magnitude \( \pi \) in the angle of the tangent vector, the continuity of \( k = |\partial \mathbf{e} / \partial \mathbf{l}| \) along with the values of the curvature's \( k \) derivatives is retained. At first sight, such a "singular" configuration of the vortex filament may seem to be physically untenable. However, one must keep in mind that the vortex filament itself is a linear singularity of a fluid. Moreover, a turbulent state of a fluid can be in general viewed as a space distribution of ruptures or discontinuities [4, 12]. Besides, owing to osculation there is no discontinuity of the one-dimensional vorticity vector field in the configuration suggested.

Figures 4c, d present possible single-filament models of the two kinds of electric charge. According to (3.2) there must be a transverse torsional stress connected with the mirror arrangement of vorticity in the disturbance. This may account for the spin of the electron or positron.

A single-filament model of two oppositely charged particles is shown in Figure 5a. Figure 5b portrays a chain-of-eddies (or "spin wave") scanning of the same system. The segment between the two disturbances (Fig. 5) corresponds to an electrostatic field. The "exterior" left and right parts of the filament (or chain of eddies) refer to an "undisturbed vacuum". A pulling apart of the peaks in Fig. 5a or ridges in Fig. 5b compels some "normally" oriented eddies to change their orientation to an opposite one. This process requires a work which just corresponds to the energy of the electrostatic field.

Absence of an electrostatic field in the case of a neutral particle may be interpreted in three dimensions as a disturbed region pierced randomly by ordinary monodirectional vortex filaments, as contrasted with more complicated shapes (see Fig. 6a with reference to Figure 4b). Occurrence of an electrical charge means that some of the vortex filaments piercing the particle are half-inverted, or "unipolar" ones (see Fig. 6b with reference to Fig. 4c), differing in this respect from the neutral particle.

The core of the elementary electrical charge may be viewed as a bundle or whisk of curved vortex filaments with a cusped point of junction (see Fig. 7 as a three-dimensional generalisation of the model shown in Figure 4c). At large distances from the core it takes the form of a hedgehog or a dandelion.

10. Interactions

In the system of two charges (Fig. 5) with a short interparticle distance the arrangement of eddies shown in Fig. 5b reflects exactly the topology of vorticity in the model of a neutral particle (Figure 4b). In
particular, the region between the two peaks (Fig. 5a) corresponds to the upper half of the loop (Fig. 4b) possessing the greatest curvature. So, already a single-filament model shows that the interaction region has a heightened curvature which grows as charges are drawing together. This effect is expressed more distinctly in three dimensions (Figure 8).

Thus, the energy of the electrostatic field, as computed from the vortex filament's curvature (see (5.1)), should be included in the self-energy of the system of charges. We can in principle resolve the total self-energy into $E_1$ and $E_2$, the energies of the charged particles taken singly, and the mutual energy $E_{12}$ of two charges. Let us generalize the motion equation (7.1) to the case of the six space coordinates $\mathbf{r}_1$ and $\mathbf{r}_2$ under consideration. For two particles of equal masses it assumes the form

$$i \frac{\partial \psi}{\partial t} = -v_1 \nabla^2 \psi - \frac{1}{\ell v_1} (e_1 + e_2 + e_{12}) \psi,$$  

(10.1)

where the Laplacian $\nabla^2$ is expressed in the variables $\mathbf{r}_1$ and $\mathbf{r}_2$, the wave function $\psi$ depends on the variables $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}$ and $t$, the energy densities $e_1(\mathbf{r}_1, t)$ and $e_2(\mathbf{r}_2, t)$ correspond to that of (7.2) for isolated particles, and $e_{12}(\mathbf{r}, t)$ is the energy density for the interaction region, which is specified by the space coordinate $\mathbf{r}$.

Take notice that a cusp $\mathbf{r}'$ is a marked point of the disturbance. This means that $\psi$ (and $e_1$ with $e_2$ as well) is in fact a function of $\mathbf{r}_1 - \mathbf{r}'_1$ and $\mathbf{r}_2 - \mathbf{r}'_2$. Hence the Laplacian $\nabla^2$ may be rewritten in coordinates of the centres $\mathbf{r}'_1$ and $\mathbf{r}'_2$ of the charged particles. According to the foregoing, everywhere except in the neighbourhood closest to the points $\mathbf{r}'_1$ and $\mathbf{r}'_2$ there must be $e_1 + e_2 \ll e_{12}$. Neglecting the terms $e_1$ and $e_2$ in (10.1) and integrating it over $\mathbf{r}$, we get the equation for the system of two charged particles:

$$i \frac{\partial \psi'}{\partial t} = -v_1 \nabla^2 \psi' + U_{12} \psi',$$  

(10.2)

where $\psi' = \int \psi \, d\mathbf{r}$ and $U_{12}$ is the interaction potential, $U_{12} \psi' = - (q \psi_1)^{-1} \int e_{12} \psi \, d\mathbf{r}$. It has been shown in a statistical model of vector corpuscles [6] (and it may also be shown in a Cosserat continuum model of the elastic substratum) that $E_{12} = \int e_{12} \, d\mathbf{r} \sim \frac{1}{|\mathbf{r}'_1 - \mathbf{r}'_2|}$ with proper account of the charge's signs. Hence $-U_{12} \sim \frac{1}{|\mathbf{r}'_1 - \mathbf{r}'_2|}$ as well. If desired, one may interchange the variables $\mathbf{r}'_1$, $\mathbf{r}'_2$ with $\mathbf{r}_1$, $\mathbf{r}_2$ in (10.2).

Certainly, electrostatics is not the only kind of interaction of the vortex sponge's disturbances. And it is not the most common one. It is well known that the nonlinear Schrödinger equation has many-soliton solutions, which can be regarded as associations of several primitive solitons. For instance, the two-soliton solution of (3.4) splits into two solitons with equal amplitudes under a persistent external perturbation [13]. In the present context, many-soliton solutions may be interpreted as a model of nuclear, or strong interaction.

11. Stochasticity

Concerning the question of the probabilistic behaviour of the microobject, it is not a matter of principle for a consistent substratum approach. Indeed, by its construction the model of a microparticle is not an isolated object but it constitutes an inalienable part of a complicated system of many degrees of freedom. The soliton of the nonlinear Schrödinger equation (as with many other kinds of solitons) is extremely sensitive to external influence. Already slight but recurring hindrances impart to the soliton's motion features of dynamical chaos. Adding to equation (3.4) a stochastic term, not altering the general form of solution, makes the soliton's motion similar to the random walk of a diffusing particle [14, 15]. In such a situation the statistical and thermodynamical mode of representation is not only appropriate but adequate. Considering the deterministic Schrödiger soliton in a classical thermostat [16] we get all necessary features of microparticle behaviour.

Thus the stochastic behaviour of a quantum object appears in fact to be none other than the statistical macroscopic manifestation (and perception) of the turbulent motion of the liquid substratum. Upon being immersed into the continually fluctuating medium (the "bath"), a stationary disturbance (Figs. 6a, 4b) of the vortex sponge emits constantly small-amplitude plane waves (disturbance waves) with random phases [14]. A dilatational component of the latter just corresponds to the gravitational field of a particle.