1. Introduction and Preliminaries

Bianisotropic media, characterized in the frequency (ω) domain by the constitutive relations [1]

\[ D(r, \omega) = \varepsilon_0 [\varepsilon(\omega) \cdot E(r, \omega) + \zeta(\omega) \cdot H(r, \omega)] \]  
\[ B(r, \omega) = \mu_0 [\zeta(\omega) \cdot E(r, \omega) + \mu(\omega) \cdot H(r, \omega)] \]

are not difficult to find in nature. Here, \( \varepsilon_0 \) and \( \mu_0 \) are the constitutive parameters of free space, \( \varepsilon_0(\omega) \) is the relative permittivity dyadic, \( \mu(\omega) \) is the relative permeability dyadic, while \( \zeta(\omega) = \xi = 0 \) represent the magnetoelectric dyadics. Materials with dyadic permittivity \( \varepsilon_0(\omega) = \varepsilon = \xi = 0 \) abound as crystals [2] and magnetoelectric dyadics for the small bianisotropic sphere are given by [8]

\[ \chi_{ee}(\omega) = 4\pi a^3 \varepsilon_0 (\varepsilon + 2\varepsilon_0) \xi^{-1} \cdot \{ \varepsilon + 3\xi \cdot A_\varepsilon^{-1} \cdot \xi^{-1} \}, \]
\[ \chi_{em}(\omega) = -12\pi a^3 \mu_0^{-1} a_0 A_n^{-1} \cdot \xi^{-1}, \]
\[ \chi_{me}(\omega) = -12\pi a^3 \mu_0 A_e^{-1} \cdot \xi^{-1}, \]
\[ \chi_{mm}(\omega) = 4\pi a^3 (\mu_0 + 2\mu_0) \xi^{-1} \cdot \{ \mu_0 - 3\xi \cdot A_n^{-1} \cdot \xi^{-1} \}. \]

On the right hand sides of (3a–d), the argument \( \omega \) has been suppressed; further,

\[ A_\varepsilon(\omega) = I - \xi^{-1} \cdot (\varepsilon + 2I) \cdot \xi^{-1} \cdot (\mu_0 + 2I), \]
\[ A_n(\omega) = I - \xi^{-1} \cdot (\mu_0 + 2I) \cdot \xi^{-1} \cdot (\varepsilon + 2I). \]

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The motivation for this work is twofold. First, the presented approach considerably generalizes previously reported work on scattering by simply moving chiral spheres [9], and may be useful for light scattering by complex aerosols. The second reason for this work is the potential use of these analyses in molecular dynamics [10], and the concurrent use in laser-based pump/probe spectroscopies for studying molecular gases [11,12]; in this latter case, we may know the polarizability dyadics of (2a,b) from other considerations [13,14].

2. Analysis

Consider the four-dimensional stationary frame $K:(r,t)$ in which all measurements are to take place. Rigidly attached to the center of the moving bianisotropic sphere is another coordinate system $K':(r',t')$; the moving frame $K'$ moves with a velocity $v$ with respect to $K$. It is assumed that at time $t = t' = 0$, the two systems $K$ and $K'$ coincide exactly. Then, the Lorentz transformations [15,16]

$$r' = r + [(\gamma - 1)(r \cdot u_v) - \gamma v t] u_v,$$
$$t' = (\gamma /c)[c t - \beta (r \cdot u_v)],$$

hold with

$$\beta = v/c, \quad c = (\varepsilon_0 \mu_0)^{-1/2}, \quad \gamma = (1 - \beta^2)^{-1/2},$$
$$v = |v|, \quad u_v = v/v.$$

The corresponding transformations for the electric and the magnetic fields are given by [15,16]

$$E' = \gamma [E + v (u_v \times B)] - (\gamma - 1)E \cdot u_v u_v, \quad (7a)$$
$$B' = \gamma [(B - \beta (u_v \times E))/c] - (\gamma - 1)(B \cdot u_v) u_v, \quad (7b)$$

where $E \equiv E(r,t), E' \equiv E'(r',t')$, etc.

2.1 Incident Plane Wave in $K$

The incident field $\{E_{inc}, B_{inc}\}$ in $K$ can be arbitrary so long as (i) its stationary source is never on or inside the scatterer, and (ii) its spectral content, as measured in $K'$ (see below), does not invalidate the applicability conditions of the long-wavelength approximation [8] inherent in the derivation of (2) and (3). Indeed, it may be convenient to work with the spatial and temporal Fourier transforms of $E_{inc}(r,t)$ and $B_{inc}(r,t)$; i.e., with a planewave spectral decomposition [17] of $E_{inc}(r,t)$ and $B_{inc}(r,t)$.

Therefore, without any particular loss of generality, the incident field in $K$ is assumed to be the plane wave

$$E_{inc}(r,t) = A \exp[i(k u_{inc} \cdot r - \omega t)], \quad (8a)$$
$$B_{inc}(r,t) = (u_{inc} \times A/c) \exp[i(k u_{inc} \cdot r - \omega t)], \quad (8b)$$

where $u_{inc}$ is a unit vector parallel to the direction of propagation, while

$$u_{inc} \cdot A \equiv 0, \quad k = \omega/c.$$

2.2 Incident Plane Wave in $K'$

The field transformations (7a, b) can be used to obtain the incident field in the moving frame $K'$. The incident field in $K'$ is still a transverse planewave specified by

$$E'_{inc}(r',t') = A' \exp[i(k' u_{inc} \cdot r' - \omega' t')], \quad (9a)$$
$$B'_{inc}(r',t') = (u'_{inc} \times A'/c) \exp[i(k' u_{inc} \cdot r' - \omega' t')], \quad (9b)$$

which expressions are in consonance with [15]. In these expressions and hereafter,

$$\omega' = \omega(1 - \beta u_v \cdot u_{inc}), \quad k' = \omega'/c, \quad (10a, b)$$
$$u'_{inc} = (\omega' /\omega)[u_{inc} + (\gamma - 1)(u_{inc} \cdot u_v) u_v - \gamma \beta u_v], \quad (10c)$$
$$A' = (\omega'/\omega) A + [\gamma \beta u_{inc} - (\gamma - 1)u_v](A \cdot u_v). \quad (10d)$$

2.3 Scattered Field in $K'$

As mentioned earlier, the frequency $\omega'$ is such that the long-wavelength approximation is to hold in $K'$; furthermore, free space is Lorentz-invariant [1]. This means that in $K'$, the scatterer is equivalent to the dipoles $p'$ and $m'$ oscillating at a frequency $\omega'$ given by

$$\begin{align*}
p' &= [x_{ee}(\omega') + (1/c) x_{em}(\omega') \cdot (u'_{inc} \times I)] \cdot A', \quad (11a) \\
m' &= [x_{me}(\omega') + (1/c) x_{mm}(\omega') \cdot (u'_{inc} \times I)] \cdot A'. \quad (11b)
\end{align*}$$

After making use of (10a–d), (11a, b) can be rewritten as

$$\begin{align*}
p' &= P \cdot A', \quad m' = M \cdot A', \quad (12a, b)
\end{align*}$$

where the two dyadics, $P$ and $M$, are defined as

$$\begin{align*}
P &= x_{ee}(\omega') \cdot \gamma (1 - \beta u_v \cdot u_{inc}) I \\
&\quad + \gamma \beta u_{inc} u_v - (\gamma - 1)u_v u_v + (1/c) x_{em}(\omega') \cdot I \\
&\quad + (\gamma - 1)(u_v \cdot u_{inc}) V - \gamma \beta V - (\gamma - 1) V \cdot (u_{inc} u_v), \quad (13a)
\end{align*}$$

$$\begin{align*}
M &= x_{me}(\omega') \cdot (u'_{inc} \times I) \cdot V - \gamma \beta V - (\gamma - 1) V \cdot (u_{inc} u_v).
\end{align*}$$
\[
M = \mathbf{z_m}(\omega) \cdot \left\{ \gamma (1 - \beta \mathbf{u}_\text{inc} \cdot \mathbf{u}_\text{inc}) I + \gamma \beta \mathbf{u}_\text{inc} \mathbf{u}_\text{inc} - (\gamma - 1) \mathbf{u}_\text{inc} \right\} + (1/c) \mathbf{z_m}(\omega) \cdot \mathbf{u}_\text{inc} \times I
\]  
(13 b)  
\[
V = \mathbf{u}_\text{inc} \times I.  
\]  
(14)

Using (2 a, b), the scattered field in \(K'\) can be obtained from the dipole moments as [8]

\[
\begin{align*}
E_{sc}(r',t') &= (\omega^2 \mu_0 \mathbf{G}(r') \cdot \mathbf{p}' + i \omega \left[ \mathbf{V} \times \mathbf{G}(r') \right] \cdot \mathbf{m}') 
\cdot \exp \left[ -i \omega t' \right],  
\tag{15a} \\
B_{sc}(r',t') &= \mu_0 \omega^2 \varepsilon_0 \mathbf{G}(r') \cdot \mathbf{m}' - i \omega \left[ \mathbf{V} \times \mathbf{G}(r') \right] \cdot \mathbf{p}' 
\cdot \exp \left[ -i \omega t' \right],  
\tag{15b} 
\end{align*}
\]

with

\[
\mathbf{G}(r') = (I + V^2 / k'^2) \left[ \exp \left( i k' r'/4 \pi r' \right) \right].  
\]  
(16)

Since the near zone in \(K'\) is small due to the small electrical size of the scatterer [18], the scattered field (15 a, b) in \(K'\) may be conveniently approximated as

\[
\begin{align*}
E_{sc}(r',t') &\approx -\left( k'^2 \exp \left[ i (k' r' - \omega' t') \right] / 4 \pi r'^3 \right) 
\cdot \left[ \varepsilon_0^{-1} \left( r' \times I \right) \cdot \left( r' \times I \right) \cdot \mathbf{p}' + c r' \left( r' \times I \right) \cdot \mathbf{m}' \right],  
\tag{17 a} \\
B_{sc}(r',t') &\approx -\left( k'^2 \exp \left[ i (k' r' - \omega' t') \right] / 4 \pi r'^3 \right) 
\cdot \left[ \left( r' \times I \right) \cdot \left( r' \times I \right) \cdot \mathbf{m}' - \varepsilon_0 \omega' r' \left( r' \times I \right) \cdot \mathbf{p}' \right],  
\tag{17 b} 
\end{align*}
\]

with \(\varepsilon_0 = \sqrt{(\mu_0 / \varepsilon_0)}\) being the intrinsic impedance of free space.

### 2.4 Scattered Field in \(K\)

To calculate \(E_{sc}(r,t)\) from (17 a, b) the Lorentz field transformations (7 a, b) have to be applied to (17 a, b) in reverse. This leads to some tedious algebra. Hence, it is convenient to define the following functions:

\[
\begin{align*}
R(r,t) &= +\sqrt{[r^2 + \gamma^2 (v_t - r \cdot \mathbf{u}_\text{inc})^2 - (r \cdot \mathbf{u}_\text{inc})^2]},  
\tag{18 a} \\
\Phi(r,t) &= -\left[ k^2 \gamma^2 (1 - \beta \mathbf{u}_\text{inc} \cdot \mathbf{u}_\text{inc})^2 / 4 \pi R^3 \right] 
\cdot \exp \left[ i k (1 - \beta \mathbf{u}_\text{inc} \cdot \mathbf{u}_\text{inc}) (R - \gamma c t + \gamma \beta r \cdot \mathbf{u}_\text{inc}) \right],  
\tag{18 b} \\
D(r,t) &= r \times I + [(\gamma - 1) (r \cdot \mathbf{u}_\text{inc}) - \gamma v_t] V.  
\tag{18 c} 
\end{align*}
\]

Then, the inversion process leads to

\[
E_{sc}(r,t) / \Phi(r,t) = \varepsilon_0^{-1} \left[ -\gamma V \cdot \left( \mathbf{V} \cdot D - \beta R I \right) + u_v (u_v \cdot D) \right] \cdot D \cdot p' 
+ c \left[ -\gamma V \cdot (R V + \beta D) + R u_v (u_v \cdot D) \right] \cdot D \cdot m'.  
\]  
(19)

Using the dyadics \(P\) and \(M\) defined in (12 a, b) and (13 a, b), (19) reduces to

\[
E_{sc}(r,t) = \Phi(r,t) 
\cdot \left[ \varepsilon_0^{-1} \left\{ -\gamma V \cdot (\mathbf{V} \cdot D - \beta R I) + u_v (u_v \cdot D) \right\} \cdot D \cdot P 
+ c \left\{ -\gamma V \cdot (R V + \beta D) + R u_v (u_v \cdot D) \right\} \cdot D \cdot M \right] \cdot A.  
\]  
(20)

Finally, the scattered magnetic field \(B_{sc}(r,t)\) in \(K\) can be readily derived using the Faraday-Maxwell relation

\[
\begin{align*}
B_{sc}(r,t) &= (1/c) r \times E_{sc}(r,t) = (1/c) (r \times I) \cdot E_{sc}(r,t) 
\tag{21} 
\end{align*}
\]

along with (20).

As a check, \(v = 0\) is set in (19); then \(\omega = \omega'\), \(k = k'\), \(r' = r\), \(t' = t\), \(A = A'\) and \(u'_\text{inc} = u_\text{inc}\) from (5 a, b) and (10 a-d). Equation (19) reduces to

\[
\begin{align*}
E_{sc}(r,t) &\approx -\left( k^2 \exp \left[ i (k r - \omega t) / 4 \pi r^3 \right] \right) 
\cdot \left[ \varepsilon_0^{-1} (r \times I) \cdot (r \times I) \cdot P + c r (r \times I) \cdot M \right] \cdot A,  
\tag{22 a} \\
M &= \mathbf{z_m}(\omega) + (1/c) \mathbf{z_m}(\omega) \cdot \left[ u_\text{inc} \times I \right].  
\tag{22 b} 
\end{align*}
\]

Thus the results of [8] for a stationary, electrically small, bianisotropic sphere are identically recovered.

### 3. Concluding Remarks

A dyadic procedure has been given for the planewave scattering response of an electrically small, bianisotropic sphere moving with a constant velocity in free space. Lorentz transformations and the polarizability dyadics of a stationary bianisotropic sphere have been used in this procedure. Specific simplifications of the presented approach are expected to be useful for electromagnetic scattering studies on aerosols and gases. Since the dyadics involved can be interpreted in terms of \(3 \times 3\) matrices, the presented method is computationally tractable. Furthermore, dyadic analysis has permitted the use of an arbitrary velocity vector \(v u_v\) and incident plane wave propagation direction \(u_\text{inc}\).