Resonance Transition of the Spatial Correlation Factor of Self-Generated Oscillations in the Postbreakdown Regime of p-Ge

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The linear correlation factor of spatially coupled nonlinear self-generated oscillations in the post-breakdown regime of p-Ge at 4.2 K is investigated. The observed results can be consistently explained in terms of a two-cell model of energy relaxation oscillations, which yields resonance transitions between correlated and anticorrelated oscillations. These are due to node-focus transitions of the fixed point of the passive cell which is slaved by the active cell.

The nonlinear spatio-temporal dynamics associated with the excitation of different spatial degrees of freedom in nonlinear dynamic systems is of great current interest. In semiconductors a variety of such phenomena has recently been studied both theoretically and experimentally in the regime of low-temperature impurity breakdown, for reviews see [1–3]. In particular, spatial correlations and crosstalk of self-generated oscillations in two electrically separated parts of a single p-Ge crystal have been investigated [4–6].

As explanation a microscopic physical model has been proposed [6] which is based upon impact ionization of shallow acceptors [7] and nonlinear energy relaxation of hot carriers [8, 9]. The crystal is represented by two spatially homogeneous identical cells which are coupled via energy exchange of the hot carriers due to the rapid propagation of phonons. In each of the cells self-generated energy relaxation oscillations can arise. If two different voltages are applied to the two cells, the model yields complex interactions of the oscillations, including quasiperiodicity and mode-locking which obeys the Farey tree ordering [10, 11].

In this paper we present theoretical and experimental investigations of p-Ge at 4.2 K which show that a sharp resonance transition between a state where the two subsystems oscillate in phase, and a state where they oscillate with a phase-lag of π, can be induced by varying the bias applied to one cell, and holding the bias at the other cell fixed. The results obtained from the microscopic two-cell model are in good agreement with our experimental findings.

In Fig. 1 the experimental configuration is shown. As described elsewhere [5] in detail, the semiconductor sample was prepared from single-crystalline p-Ge with an acceptor doping concentration of about $10^{14}$ cm$^{-3}$ and typical dimensions of about $0.25 \times 2 \times 8$ mm$^3$. Four ohmic Al contacts (hatched areas) evaporated onto one of the two largest surfaces divide the Ge crystal into three separate subsections. To probe the spatial correlation between different parts of the same crystal, the bias voltages $V_1$ and $V_2$ were applied to the two outer subsystems 1 and 2, respectively, while the potential difference between the inner contact electrodes was always kept zero. In this way, no charge carriers could be transported across the intermediate subsection, such that lattice heat conductivity remained as the only important coupling mechanism in between. Our experimental system then consisted of two electrically separated and diffusively coupled subsections, each of which could electrically be driven into impact ionization breakdown capable of generating spontaneous current oscillations. The resulting sample currents $I_1$ and $I_2$ were measured using load resistances (1 Ω) connected in series to the correspond-
Fig. 1. Scheme of the experimental set-up.

The sample investigated was kept at the liquid-helium temperature of 4.2 K and carefully shielded against external irradiation (visible and far infrared).

The model is given by the following set of balance equations [9] for the hole concentrations \( p_i \) and the mean energies of the holes \( E_i \) in the two cells \((i = 1, 2)\):

\[
\dot{p}_1 = f_p(p_1, E_1),
\]

\[
\dot{E}_1 = f_p(p_1, E_1) + D(E_2 - E_1),
\]

\[
\dot{p}_2 = f_p(p_2, E_2),
\]

\[
\dot{E}_2 = f_p(p_2, E_2) + D(E_1 - E_2)
\]

with

\[
f_p(p, E) = X(E)p(N^*_A - p) - T^*(E)[p(N_D + p) - p_0(N^*_A - p)],
\]

\[
f_E(p, E) = e\mu\delta^2 = -(E - E_0)/\tau_c
\]

\[
- E_{th} X(E)(N^*_A - p) - E f_p(p, E)/p,
\]

where the impact ionization coefficient \( X \) and the capture coefficient \( T^* \) are modelled by the phenomenological functions

\[
X(E) = X_0 \exp \left( -\frac{E_{th}}{E} \right) \left( \frac{E}{E_{th}} \right)^{-\sigma},
\]

\[
T^*(E) = \begin{cases} T_{th}^*, & E < E_r, \\ T_{th}^* [(s + 1) - s(E_r/E)](E/E_r)^{-\sigma}, & E \geq E_r \end{cases}
\]

with positive parameters \( X_0, E_{th}, r, T_{th}^*, E_r, s \), \( N^*_A = N_A^* - N_D \), \( N_A \), and \( N_D \) are effective doping concentration, acceptor and compensating donor density, respectively, \( \mu \) is the hole mobility, \( \delta_i \) the electric field applied to cell \( i \), \( E_0 \) the electron energy in thermal equilibrium with the lattice, \( \tau_c \) the energy relaxation time, \( E_{th} \) the impact ionization threshold energy, and \( p_0 \) the thermal equilibrium hole concentration for a Fermi level coinciding with the acceptor level. The \( p_0 \)-term in (5) describes thermal ionization of holes [1]. As convenient control parameters we introduce the dimension-
less net absorbed power per carrier in each cell:

\[ P_i = \frac{(e \mu \delta_i^2 \tau_e + E_0)}{E_{th}}, \quad i = 1, 2. \]  

The material parameters used in the following are: 
\( N_A = 10^{14} \, \text{cm}^{-3}, \quad N_D = 10^{12} \, \text{cm}^{-3}, \quad p_0 = 3 \times 10^{-11} \, \text{cm}^3 \text{s}^{-1}, \quad \tau_e = 10^{-7} \, \text{s}, \quad E_{th} = 10 \, \text{meV}, \quad T_0 = 3 \times 10^{-5} \, \text{cm}^3 \text{s}^{-1}, \quad X_0 = 10^{-6} \, \text{cm}^3 \text{s}^{-1}, \quad E_c = 10 \, E_{th}, \quad r = 0.5, \quad s = 1.5, \quad D = 0.05/\tau_e, \) corresponding to p-Ge at 4.2 K [9].

Figure 2 depicts a sequence of time series and phase portraits obtained from (1)–(8) by varying \( P_1 \), and holding \( P_2 = 11 \) fixed. The time series and phase portraits in Figs. 2(a), (e) and (b), (d) reveal an approximately linear anticorrelation or correlation between \( p_1 \) and \( p_2 \), i.e., the two cells oscillate (a), (e) with a phase-lag of \( \pi \), and (b), (d) in phase. Such behaviour was indeed observed (Figure 3). A quantitative mea-
The dependence of $F$ upon $P_1$ shows five different regimes: (i) anticorrelation of $p_1$ and $p_2$ ($F \approx -1$) for $0 \leq P_1 \leq 6$, (ii) correlation ($F \approx +1$) for $6 \leq P_1 \leq 10$, (iii) no linear correlation ($F \approx 0$) for $10 < P_1 < 13$ (except for the symmetric case $P_1 = P_2 = 11$), (iv) correlation for $13 < P_1 < 14$, and (v) anticorrelation for $14 < P_1 \leq 17$. Between regimes (i) and (ii), or (iv) and (v), there occur...
very sharp transitions. Experimentally, the linear correlation factor between the currents $I_1$ and $I_2$ has been determined as a function of the voltage $V_1$ applied to subsystem 1 for fixed bias voltage $V_2$. Figure 4 shows that the agreement between theory and experiment is excellent.

The discontinuous behaviour of the linear correlation factor can be physically understood by noting that the five regimes (i)--(v) correspond to five qualitatively different states of the dynamic system (1)--(4). A linear stability analysis of its fixed point reveals that it undergoes a characteristic sequence of transitions [10, 11]. While $P_2$ is fixed in a range where the system 2 would be without coupling ($D=0$) in an actively oscillating state (stable limit cycle), $P_1$ is varied in a range where without coupling the subsystem 1 would successively display (i) a stable node, (ii) a stable focus, (iii) a stable limit cycle, (iv) a stable focus, (v) a stable node. The coupling does not essentially affect this behaviour, so that we have two "active" subsystems in the regime (iii), and one "active" and one "passive" subsystem elsewhere. In the regime (iii) the linear correlations break down ($F\approx0$), and complex mode-locking structures and quasiperiodicity occur [11]. In the other regimes the passive subsystem 1 is slaved by the active system 2, and linear (anti-)correlations occur. In the regimes (i) and (v) the passive subsystem behaves as an overdamped oscillator (stable node), and follows the active subsystem with phase-reversal ($F\approx-1$). In the regimes (ii) and (iv) the passive subsystem is a damped oscillator (stable focus) with an intrinsic frequency mostly larger than that of
the active system, and oscillates in phase \((F \approx 1)\), like a periodically driven, damped linear oscillator below resonance.

Thus the sharp transitions between anticorrelation \((F = -1)\) and correlation \((F = +1)\) represent a resonance phenomenon associated with a node-focus transition of the passive subsystem. Finally, it should be pointed out that the linear correlation factor \(F\) becomes meaningless if the oscillations in the two subsystems have a phase lag other than \(\pi\) or zero, or if they are nonlinearly correlated, for instance, in the regime (iii) of two active oscillators. A characteristic nonlinear measure, which is particularly sensitive to correlations, and which is meaningful both in the active and in the passive regimes, is the bit-number variance of the asymptotic invariant density of the dynamic system [12], which was originally introduced in nonequilibrium thermodynamics as a generalization of the specific heat [13]. An experimental investigation of this quantity is currently in preparation.

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