The X-ray Structure Factor of Liquid Glycerol

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The X-ray scattering intensity from highly viscous, liquid glycerol has been measured using MoKα-radiation (0.2 < q Å-1 < 10) between 250 K and 350 K. The corrected spectra were Fourier transformed to obtain the atomic radial distribution function.

For increasing temperatures one observes an increase of the X-ray intensity before the first structure maximum, and a broadening and a shift to lower-q-values of the main peak, while the peak height remains nearly constant.

The temperature dependent structure factors are compared with a model calculation, which in a simple way gives the correct compressibility limit. From the fits to the structure maximum the existence of orientational correlations is inferred.

1. Introduction

The spatial arrangement of nonspherical molecules forming a liquid is often described by independent center of mass and orientation correlation functions. The latter will depend on the structure of the molecule and on the interaction potential. Both correlation functions are, due to the thermal motion of the molecules, time dependent and experiments with high energy resolution like NMR, light-, neutron- and Mössbauer-scattering are suited to measure aspects of the time evolution of the correlation functions. For liquid glycerol, C3O3H8, several measurements of this kind exist (see [1, 2, 3, 5] and references cited therein). This liquid is highly associated, supercools easily, and the range of values which the diffusion coefficient assumes [4] extends continuously from D ≈ 10-13 cm²/sec (at the glass transition temperature ~ 180 K) to values characteristic for normal liquids (> 400 K). If one tries to derive a translational energy E_tr from the translational diffusion, even a very rough estimate shows that E_tr < kT below say 350 K. For details see e.g. the quasielastic neutron and Mössbauer scattering results [2, 3]. On the other hand the quasielastic part of the scattered intensity decreases with increasing T, similar to the Debye-Waller factor [2, 3]. These facts suggest an attempt to treat scattering from glycerol in its liquid phase like scattering from an amorphous solid with phononlike vibrations.

To our knowledge so far no X-ray structure investigations of liquid glycerol were reported despite the fact that many dynamical studies are available. We have therefore measured the X-ray scattering for momentum transfer q = 0.2 to 10 Å-1. The analysis of the data will not only have to deal with the temperature dependence of the molecular structure, but also with the effect of (acoustic) phonons on the intensities, with the possibility of a conformational change of the molecules when one goes from the crystalline to the liquid state, and with the effect of orientational correlations on the structure factor.

After describing the experiment in Sect. 2 and the theoretical background in Sect. 3, we compare in Sect. 4 the results with a simple model-structure-function which takes into account the effects just mentioned.

2. Experiment and Scattering Intensities

2.1. Experimental Set-up

The experiments were performed on an X-ray 0-0-goniometer which has been described elsewhere [6]. The scattering chamber consists of brass with two parallel beryllium windows (diameter 30 mm) 2 mm apart. It was filled with dry glycerol and operated in Laue transmission geometry.
Heating and cooling of the sample as well as the protection against humidity was the same as in a former Mössbauer experiment [2].

Measurements were performed using MoK\textsubscript{\alpha} radiation ($\lambda = 0.711$ Å) between $\theta = 0.6^\circ$ and $35^\circ$ in steps of $0.05^\circ$. The sample temperatures were $T = 250$ K, 275 K, 300 K, 325 K, 350 K.

2.2. Corrections and Normalization

The measured intensity distributions were corrected for empty container scattering, absorption and polarization. The correction for Compton-scattering was performed as follows: the necessary wavelength profile of the Compton inelastic scattering was calculated for several angles [7] and multiplied by a rectangular step function of width $\Delta\lambda$, given by the wavelength acceptance curve of the monochromator. The resultant profile was then integrated over the wavelength and this value gave the Compton scattering intensity accepted by the monochromator at this scattering angle. In order to obtain the scattered intensity per molecule (in electron units (eu)), the Compton intensity of all atoms of the glycerol molecule was added up

$$I^{\text{comp}}(\theta) = \sum_{r} \int_{\lambda_{i}}^{\lambda_{f}} I^{\text{comp}}(\theta, \lambda) d\lambda.$$  \hspace{1cm} \text{(1)}$$

The X-ray atomic scattering factors $f_{r}(q)$ were taken from Cromer and Weber [8] with the dispersion corrections from Dauben and Templeton [9].

2.3. Experimental Results

Figure 1a shows the normalized intensity distributions $I_{\text{ex}}(q)$ for $T = 250$ K and 350 K divided by the incoherent molecular scattering intensity $A_{\text{in}} = \sum_{r} f_{r}^{2}(q)$ (= 308 eu at $q = 0$). With increasing temperatures one observes an increase of the intensity on the left hand side of the structure maximum, a shift of the latter to lower $q$-values, and a broadening of the peak itself. An increase at very low $q$ (down to 0.2 Å\textsuperscript{-1}) is also observed. At this point the hydrodynamic region might be attained where an increase in intensity is partly due to an increase of the isothermal compressibility $\chi_{T}$ according to

$$I(q = 0) = 2\pi k_{\text{B}} T \chi_{T} \left( \sum_{r} f_{r}(0) \right)^{2},$$  \hspace{1cm} \text{(2)}$$

where $n$ is the molecular number density. The low $q$-range is presented separately in Figure 4.

In Fig. 1b the one dimensional Fourier transform of the curve $I_{\text{ex}}(q) - \sum_{r} f_{r}^{2}(q)$ is displayed, which is $4\pi r^{2}(q - q_{0})$, $q_{0}$ being the electron density averaged over the sample. These functions represent the average electronic density at distance $r$ from the center of mass of a mean atom, where the electronic density distribution of this mean atom appears as an additional broadening of the peaks of the spatial distribution functions.

The fact that the 350 K curve in Fig. 1b shows more oscillations than the 250 K curve is due to the shorter $q$-range measured.

3. The Scattering Law

We shall assume that the liquid consists of one type of rigid molecules only (one conformation), and we separate the scattering intensity into self and distinct parts, $I(q) = I_{s}(q) + I_{d}(q)$. The center of mass (CM) of molecule $i$ is $R_{i}$. The atoms have positions $r_{i}$ with respect to a molecular frame of reference, which itself has orientation $w_{1} = (\theta_{1}, \varphi_{1}, \psi_{1})$ with respect to the laboratory frame of
reference. Then the molecular scattering amplitude of molecule 1 is

\[ A(q, w_1) = \sum_f f(q) \exp[i q (R_1 + D(w_1) r_f)], \]

(3)

where \( D(w_1) \) is the rotation matrix connecting the reference frames.

The self part of \( I(q) \) is

\[ I_s(q) = \langle A^2(q, w) \rangle = \sum_{\mu} \sum_{\nu} f_\mu f_\nu \sin(q r_{\mu\nu})/q r_{\mu\nu} \]

\[ = \langle A(q, w) \rangle^2 + \sum_{l=1}^{\infty} \sum_{\mu} f_\mu f_\nu (2l + 1) \]

\[ \cdot j_l(q r_{\nu}) j_l(q r_{\mu}) P_l(\cos \theta_{\mu\nu}). \]

(4)

The averaging over orientation \( w \) is denoted by brackets. \( \theta_{\mu\nu} \) is the angle between \( r_{\mu} \) and \( r_{\nu} \) within one molecule, \( r_{\mu\nu} = |r_{\mu} - r_{\nu}| \), \( j_l(q r) \) are the spherical Bessel functions, and \( P_l(\cos \theta_{\mu\nu}) \) the Legendre polynomials.

The distinct part becomes

\[ I_d(q) = \int A(q, w_1) A(q, w_2) \cdot F_d(q, w_1, w_2) \, dw_1 \, dw_2, \]

(5)

where \( F_d(q, w_1, w_2) \) is the Fourier transform of the distinct correlation function which gives the probability to find a molecule at \( R_2 \) with orientation \( w_2 \), given that there is a molecule at \( R_1 \) with orientation \( w_1 \).

If there is no orientational correlation, the integration in (5) can be performed, \( F_d(q, w_1, w_2) \) becomes simply the CM-correlation function \( F_d(q) \), and

\[ I(q) = \langle A^2(q) \rangle + \langle A(q) \rangle^2 F_d(q). \]

(6)

One may look at (6) as the first term in an expansion

\[ F_d(q, w_1, w_2) = F_d(q) + F_{OR}(q, w_1, w_2). \]

Restricting ourselves to the term (6), we have to discuss the molecular structure, \( A \), and the CM-CF, \( F_d(q) \). The effect of the next term, the orientational correlation function (OR-CF), \( F_{OR}(q, w_1, w_2) \), will be considered in Section 4. While the present simple treatment derives from [2], more elaborate formulations have been presented e.g. by Zeidler et al. [10].

### 3.1. Molecular Structure

A first guess for \( A(q, w) \) is the molecular structure in crystalline glycerol as measured by Koningsveld [11] using X-rays. The carbon and oxygen positions which he found are given in Table 1. This is the so called \( \alpha \alpha \)-conformation with no intramolecular hydrogen bonds possible. The shape of the molecule is rather elongated and the association occurs via intermolecular hydrogen bonds.

In order to study the influence of the molecular structure in the liquid, we have calculated \( A \) for several conformations. Looking at the isolated molecule, a conformational change to a more compact form can easily be performed by rotating an oxygen \( O_3 \) (and/or \( O_1 \) around the \( C_2-C_3 \) (and/or \( C_2-C_1 \) axis, leading to intramolecular bonding. The results for \( \langle A^2 \rangle \) and \( \langle A \rangle^2 \) are presented in Figure 2. Normalized to the sum of the atomic scattering intensities \( \sum f_\mu^2 \), we show the molecular intensity averaged over orientation, \( \langle A^2 \rangle \), as well as the intensity due to the nonspherical shape of the molecule \( \langle A^2 \rangle - \langle A \rangle^2 \). The latter is equivalent to the sum of all nonzero moments if one expands the

![Fig. 2. \( \langle A^2 \rangle - \langle A \rangle^2 \)/\( \sum f_\mu^2 \), deviation of the molecular formfactors \( \langle A^2 \rangle \) from the mean values \( \langle A \rangle^2 \) for two conformations (thick lines). Thin lines correspond to \( \langle A^2 \rangle \). Dashed lines: elongated molecular shape from crystalline structure. Solid lines: compact shape by rotating \( O_3 \) about 180° around \( C_2-C_3 \)-axis (see Table 1, [11]). \( A_{m2} = A^2/\sum f_\mu^2 \).](image-url)
scattered intensity into spherical harmonics [1], see (4). The effect of a change of the molecular shape on the total structure function is rather drastic around $q = 1 \text{Å}^{-1}$ as can be concluded from Figure 2. However, the steep rise of $F_d(q)$ falls in the same region, and the OR-CF is also important there.

3.2. Center of Mass Correlation and Thermal Diffuse Scattering

For low $q$, homogeneous systems are rather transparent ($F_d(q) \approx -1$) except for the long wave-length density fluctuations due to thermally excited phonons which are responsible for the compressibility limit, cf. (2). This limit is easy to verify if one uses the scattering law for a crystalline powder and evaluates it within the zeroth Brillouin zone (BZ). To the extent that longitudinal sound waves are the main contributors to $I(q \to 0)$, this calculation should also be valid for the center of mass scattering of glycerol $F_d(q)$ up to $q$ about half the structure maximum position.

One gets (up to two phonon processes)

$$F_d(q) + 1 = S(q) = \frac{e^{-2M}}{\alpha} \left[ \delta(q/\alpha) + \frac{k_B T}{m v^2} \frac{q^2}{2} \int_{BZ} \frac{\delta(g + g_1)}{g_1^2} dq_1 + \alpha^2 \left( \frac{k_B T}{m v^2} \right)^2 q^4 \right]$$

with $v =$ velocity of sound; $V_z =$ volume of primitive cell; $g =$ phonon wave vector; $m =$ molecular mass; $\alpha = V_z/(2\pi)^3$. For an extensive discussion and the higher terms, see e.g. [12]. We shall omit the forward scattering due to $d(q)$.

The Debye Waller exponent is small in the first BZ:

$$2M = q^2 < T^2 > = \frac{3 k_B T V_z}{m v^2} q^2 \left( \frac{d}{d^3} \right)^2 + a^2 T l \left( \frac{d}{d^3} \right)^2$$

One then has to a good approximation for $q \to 0$

$$S(q) = \frac{k_B T}{m v^2} \left[ 1 + \frac{k_B T}{m v^2} \frac{3 \pi^2}{8} q^3 \right].$$

From

$$c_v = \frac{N E}{V^2 c_T} = \frac{N E}{V^2 c_p} = \frac{1}{m v^2}$$

with $p = E \neq V$ the pressure; $E = N m v^2$ the mean internal energy; $c_p \approx c_v$ the specific heats, one recovers from (8) for the limit $q = 0$.

$$S(q = 0) = \frac{q_n \chi_T k_B T}{m v^2}.$$  

This form leads back to (2).

From (7) it is obvious that in the absence of small angle scattering due to clustering the low $q$ region of the scattering law produces purely inelastic scattering (Brillouin scattering). The $q$ independence of the one phonon scattering stems from the high temperature limit adopted throughout. A more accurate evaluation produces a weak increase of this part with $q$.

3.3. The Total Structure Function, Elastic and Inelastic Contribution

From the model and assumptions described, the following expression for $I(q)$ was derived for...
comparison with the experimental results:

\[ I(q) = S(q)[\langle A \rangle^2 + A_{OR}] \\
+ S_{s}(q)[\langle A^2 \rangle - (\langle A \rangle^2 + A_{OR})] \quad (12) \]

with

\[ A_{OR} = \int A(q, w_1)A(q, w_2) \\
\cdot F_{OR}(q, w_1, w_2)dw_1dw_2/F_a(q), \]

\[ S(q) = S_0(q) + \sum_{m=1}^{\infty} S_m(q) = S_{el} + S_{inel}. \]

and

\[ S_{s}(q) = e^{-q^2<x^2>} + (1 - e^{-q^2<x^2>}) \]

the self term of the center of mass scattering law.

The terms \( S_m(q), m = 0, 1, ... \) accounts for the phonon contributions (zero phonon, one phonon etc.). The OR-CF was set to zero for the fits.

4. Comparison with Experiment and Discussion

The result of the comparison of our simple model-function (12) with the measured structure factor is given in Fig. 3 for \( T = 250 \) K and in Fig. 4 for the low \( q \)-range for all temperatures. The input data for the inelastic contribution to \( S(q) \) are collected in Table 2. We used the velocity of sound computed from the isothermal compressibility [13]. The parameters \( a, \Delta \) were obtained by selecting by eye the curves \( I(q) \) which fitted best the structure maximum. They are \( a = 7.15 \, \text{Å} \pm 0.02, \Delta = 0.75 \, \text{Å} \pm 0.05 \); the errors quoted are rough estimates, essentially independent of the molecular configuration. The next neighbour distance is \( d = a/\sqrt{2} \).

In the low \( q \)-range (\( q < 0.5 \, \text{Å} \)) the inelastic intensity is dominant. The extrapolated values of the measured intensities at \( q = 0 \) meet the compressibility values within 20\%, see Figure 4. This result would not be achieved in the case of uncorrelated vibrations (Einstein oscillators) since finite energy modes produce zero intensity at \( q = 0 \).

At the structure maximum one gets also a small inelastic peak. Such a peak is very pronounced in crystals, but due to the distribution of lattice constants which we use, the phonon structure is also

Table 2. Input data for calculation of inelastic scattering. Compressibility \( \chi_T \) [13] and sound velocity \( v_s \) derived from it. \( v_0 \) [14] is from ultrasonic measurement (low frequency part).

<table>
<thead>
<tr>
<th>( T[K] )</th>
<th>( \chi_T[10^{-6} \text{ atm}^{-1}] )</th>
<th>( v_s[\text{m/s}] )</th>
<th>( v_0[\text{m/s}] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>17.0</td>
<td>2110</td>
<td>2010</td>
</tr>
<tr>
<td>275</td>
<td>19.0</td>
<td>2035</td>
<td>1950</td>
</tr>
<tr>
<td>300</td>
<td>21.0</td>
<td>1960</td>
<td>1890</td>
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<tr>
<td>325</td>
<td>23.0</td>
<td>1876</td>
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<tr>
<td>350</td>
<td>25.0</td>
<td>1830</td>
<td>1780</td>
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</tbody>
</table>
broadened. At still higher \( q \) the inelastic scattering
derived from correlated or uncorrelated vibrations
differs very little. The mean squared vibrational
amplitude \( \langle x^2 \rangle \) was 0.02 Å\(^2\) at 250 K and 0.04 Å\(^2\)
at 350 K, which is much lower than derived from
quasielastic scattering [2].

The distance between next neighbour molecules
increases from 5.06 Å at 250 K to 5.16 Å at 350 K,
an increase of \((2 \pm 0.5)\%\) in good agreement with
increases from 5.06 Å at 250 K to 5.16 Å at 350 K,
which is much lower than derived from
for all temperatures.

The mean squared vibrational
of the next neighbour distance for all temperatures.

The distribution parameter \( \delta \) is about \((10 \pm 2)\%\)
of the next neighbour distance for all temperatures.

From the overall fit to the experimental curves
two main features should be discussed:

If one uses for the molecular form factor the
crystalline, elongated conformation, Fig. 3a, the
computed total intensity is too high for \( 0.5 < q \AA \)
and too low for \( 1.8 < q \AA < 2.8 \), the region of
the second experimental structure peak. In Fig. 3b
the compact conformation of the molecule was
assumed. The discrepancy on the left side of the
main peak is removed but the indication of a second
peak near 2.7 Å\(^{-1}\) has disappeared.

The differences at the higher q-range \( q > 3 \AA^{-1}\)
between both conformations are not considered
significant. The deviation of the third peak position
may arise from an error in the average intramolec-
ular distances. Due to the limited q-range actually
measured, the normal access to the molecular form-
factors by fitting only the high q-range [15] is for
the present case of no value.

We turn now to orientational correlations and
their interplay with possible molecular shapes. In
Fig. 3 the term \( S(q) \cdot \langle A \rangle^2 \), is — according to our
assumptions so far — the only term sensitive to
nearest neighbours. (It is constructively interfering
scattering from the average spherical electron
density, i.e. the \( l=0 \) term in (4), of neighbour
molecules.) This term drops sharply for both con-
formations at \( q > 1.9 \AA^{-1} \), because \( \langle A \rangle^2 \) becomes
very small (see (4) and Figure 2). This has the
consequence that the second peak of \( S(q) \), which
occurs at \( q = 2.9 \AA^{-1} \), does not show up in \( S(q) \cdot \langle A \rangle^2 \).

If one now involves orientational correlations
also, the following interpretation of the experiment
seems to be possible:

Except for the zeroth moment all higher scat-
tering moments, \( \langle A^2 \rangle - \langle A \rangle^2 \), produce an appreci-
able intensity for \( q \)-values > 1 Å\(^{-1}\), see Figure 2. If
the \( A_{OR} \) are different from zero this has relatively
little effect on the first term of (12) for \( 0.5 < q \AA
< 1 \), because there \( \langle A \rangle^2 \ll A_{OR} \). On the other hand,
the second term would be remarkably reduced by
\( A_{OR} \) because \( \langle A^2 \rangle - \langle A \rangle^2 \) is small, perhaps com-
parable to \( A_{OR} \). The necessary depression of the
calculated curve in Fig. 3a at \( q \approx 1 \AA \) can thus be
achieved either by the compact shape (as discussed
above) or by including orientational correlation.
On the right hand side of the main peak our model
function is so far fully determined by single molec-
ular scattering \( (S(q) \cdot \langle A \rangle^2 \approx 0) \). It appears not
unreasonable to expect that \( A_{OR} F_d \) has an oscil-
ating shape similar to \( F_d \). Such a peak, if situated
at \( q = 2.6 \AA^{-1} \), would offer an explanation for the
experimental peak at this \( q \): The higher scattering
moments are practically constant beyond 2 Å\(^{-1}\),
see Fig. 2, and the intensity will thus reproduce the
\( q \)-dependence of \( A_{OR} \) through the first term of (12).

The deviations of the model function, Fig. 3a,
from the experimental structure factor on both
sides of the main structure peak can thus be cor-
corrected by including orientational correlations,
whereas starting with Fig. 3b, one would trade an
improvement at \( q = 2.6 \AA^{-1} \) for a worsening of the
agreement at 1 Å\(^{-1}\). We thus conclude that the
elongated molecular shape and non negligible
orientational correlations are the more plausible
situation.

5. Summary

We have presented a set of X-ray measurements
on liquid glycerol and compared the data with a
simple model for an amorphous structure which is
including acoustic phonons. The intensity extra-
opolated to \( q = 0 \) is in good agreement with the iso-
thermal sound velocity. The model allows a straight-
forward calculation of the elastic and inelastic
scattered intensity (effective Debye-Waller factor)
for the whole q-range.

The experimental scattering intensity displays a
lot of details which we have tried to interprete
using the model mentioned. From a consideration
of the interrelated effects of molecular formfactors
(average conformation of the molecule in the liquid),
center of mass correlations, and orientational cor-
relations it was concluded that the crystalline con-
formation of the molecule appears to be predomin-
antly extended.
nant in the liquid phase also and that orientational correlations seem to have a peak at $q \approx 2.6 \text{Å}^{-1}$.

The use of a crystalline lattice function with a distribution of lattice constants for the molecular center of mass correlation function is certainly a crude assumption for higher $q$-values, while it is reasonably correct in the hydrodynamic limit. It should be understood as a simple means for an estimate of the thermal diffuse scattering from amorphous structures to which strongly supercooling liquids may be assigned.

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