Cherenkov Losses of a Relativistic Neutron in a Conducting Medium

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Expressions are derived for the combined Cherenkov and Ohmic losses of a relativistic magnetic dipole moving in a (1) strongly and (2) weakly conducting medium.

1. Introduction

The Bohr losses of electrically charged particles in cloud chambers, photographic plates, scintillation counters etc. were summarized by Price, and their Cherenkov losses in a variety of media by Jelley. A unified treatment of the Bohr and Cherenkov losses was first given by Fermi. The combined Cherenkov and Ohmic losses of a charged particle in a conducting medium have been recently studied by us in an earlier paper. The present study is intended to investigate the companion problem of a relativistic neutron and thus extend the earlier works of Frank, Eidman, Ginzburg and Balazs to conducting media. Apart from their intrinsic theoretical interest, studies of radiation in conducting media in motion have acquired renewed significance by virtue of their astrophysical and spatial significance.

In view of the duality symmetry inherent in Maxwell's equations in the absence of conductivity, the problem of a neutron can be trivially solved by a mere inspection of the results for a charge. This however is no longer the case in the electrically conducting medium, since it does not possess a magnetic counterpart; and the problem has thus to be examined de novo. An investigation from the rest frame of the medium suffers from the drawback that a magnetic dipole appears to acquire an additional electric dipole moment by virtue of its motion. By the same token, however, the rest frame of the particle also entails its own complications stemming from an involved constitutive matrix of a moving medium, which now acquires a uniaxial magnetoelectric character. Added to this the conductivity of the medium worsens the situation, since a conducting medium loses its neutrality and appears to be charged when set in motion in an electric field. All these difficulties are however offset by the enormous simplification that the fields become static in the rest frame of the particle, which permits the losses to be derived from a three-fold Fourier integral.

We therefore prefer to work in the rest frame of the neutron and obtain the losses as the work done on it by the fields set up by the polarization of the medium. We first derive the losses of a magnetic monopole and from these deduce the losses of a neutron whose magnetic moment can be looked upon as due to a juxtaposition of two monopole moments. Neat expressions could be obtained for the two realistic limiting cases of a strongly and a weakly conducting medium.

2. Fourier Synthesis of the Monopole Fields

Let the point monopole (m) move with a uniform velocity $\beta c$ along the $x_1^0$ axis in a homogeneous isotropic neutral conducting medium, with real values of the permittivity ($\varepsilon$), permeability ($\mu$) and conductivity ($\sigma$). In the rest frame of the particle, the Maxwell’s equations take the form

$$\nabla \times E = 0, \quad \varepsilon_{m} \nabla \times B = 0,$$

$$\nabla \times B = \frac{2}{c} [J_{\mu}(x_1) + \beta_{\mu} q_0(x_1)],$$

where $J_{\mu}$ is the conduction current density and $\beta_{\mu} q_0$ is the convection current density due to the apparent charge density $q_0$ in the relativistically moving conducting medium. The static fields can be resolved into their Fourier components as

$$D_{\mu}(x_1) = \int D_{\mu} \exp i k_1 x_1 d^3 k,$$

etc.
The constitutive relations of the moving conducting medium can be obtained by a Lorentz transformation of the covariant material tensors $T_{ijkl}$ and $\omega_{ijk}$ that connect the Fourier components of the induction tensor $H_{ij}$ and the current density four vector $J_i$ respectively with the field tensor $F_{ij}$. These are then given by

$$D_1 = \varepsilon_1 E_1, \quad D_2 = \varepsilon_2 E_2 + \xi B_3, \quad D_3 = \varepsilon_2 E_3 - \xi B_2, \quad H_1 = \lambda_1 B_1, \quad H_2 = \lambda_2 B_2 + \xi E_3, \quad H_3 = \lambda_2 B_3 - \xi E_2, \quad J_1 = \eta E_1, \quad J_2 = \eta E_2 + \Psi B_3, \quad J_3 = \eta E_3 - \Psi B_2,$$

(3)

and

$$\omega = -\Psi E_1,$$

where

$$\varepsilon_1 = \varepsilon, \quad \varepsilon_2 = \frac{r^2}{\mu} (n^2 - \beta^2), \quad \lambda_1 = \frac{1}{\mu}, \quad \lambda_2 = \frac{r^2}{\mu} (1 - \beta^2 n^2), \quad \xi = \frac{\beta r^2}{\mu} (n^2 - 1), \quad \eta = \sigma r, \quad \Psi = \sigma \beta r, \quad n^2 = \varepsilon \mu.$$

Substituting (2) and (3) in (1), we obtain

$$H_1(x_i) = \frac{i m}{8 \pi^2} \int \frac{d^3 k}{\mu} \frac{k_1 \exp i k_1 x_i}{k_2^2 + k_3^2 - \alpha^2 k_1^2 - 2 i \chi k_1},$$

(4 a)

and

$$H_2(x_i) = \frac{m}{4 \pi^2} \int \frac{d^3 k}{\mu} \frac{\chi \exp i k_1 x_i}{k_2^2 + k_3^2 - \alpha^2 k_1^2 - 2 i \chi k_1},$$

(4 b)

where $\alpha^2 = r^2 (\beta^2 n^2 - 1)$ and $\chi = (\sigma \beta r \mu) / c$. In the limit of a non-conducting medium, $\chi \rightarrow 0$, (4 b) vanishes and (4 a) becomes the dual of the $E_1$ for an electric charge.

### 3. Energy Losses of a Magnetic Monopole

Since $H_1$ is unaltered by the special Lorentz transformation, we obtain the energy loss of a magnetic monopole as

$$W = -\frac{i m^2}{8 \pi^3} \int_{-\infty}^{\infty} \frac{d^3 k_1}{\mu} k_1 \int_{-\infty}^{\infty} d k_2 \int_{-\infty}^{\infty} d k_3 \frac{d k_3}{k_2^2 + k_3^2 - \alpha^2 k_1^2 - 2 i \chi k_1}$$

$$+ \frac{m^2}{4 \pi^3} \int_{-\infty}^{\infty} \frac{d^3 k_1}{\mu} \int_{-\infty}^{\infty} d k_2 \int_{-\infty}^{\infty} d k_3 \frac{d k_3}{k_2^2 + k_3^2 - \alpha^2 k_1^2 - 2 i \chi k_1},$$

(5 a)

and in a weakly conducting medium as

$$W = \frac{-m^2}{4 \pi^2} \int_{-\infty}^{\infty} \frac{d^3 k_1}{\mu} \int_{-\infty}^{\infty} d k_2 \int_{-\infty}^{\infty} d k_3 \frac{d k_3}{k_2^2 + k_3^2 - \alpha^2 k_1^2 - 2 i \chi k_1}$$

and in a strongly conducting medium as

$$W = \frac{-m^2}{4 \pi^2} \int_{-\infty}^{\infty} \frac{d^3 k_1}{\mu} \int_{-\infty}^{\infty} d k_2 \int_{-\infty}^{\infty} d k_3 \frac{d k_3}{k_2^2 + k_3^2 - \alpha^2 k_1^2 - 2 i \chi k_1}.$$

(6)

and in a weakly conducting medium as

$$W = \frac{-m^2}{4 \pi^2} \int_{-\infty}^{\infty} \frac{d^3 k_1}{\mu} \int_{-\infty}^{\infty} d k_2 \int_{-\infty}^{\infty} d k_3 \frac{d k_3}{k_2^2 + k_3^2 - \alpha^2 k_1^2 - 2 i \chi k_1}.$$

In the above expressions, $\tau$ is the relaxation time of the conducting medium and $\omega_h$ and $\omega_l$ are the two limiting frequencies characteristic of the medium below and above which the medium can be classed as strongly or weakly conducting respectively.

For an insulating medium $\tau \rightarrow \infty$, and the second term in (7) vanishes, leaving the Cherenkov losses given by the first term. It is thus obvious that the two terms in (7) refer respectively to the Cherenkov and Ohmic losses. Such a division is absent from (6) as is to be expected, for, in a strongly con-
ducting medium, any Cherenkov emission that takes place is rapidly absorbed and the entire loss partakes an Ohmic character.

4. Energy Losses of a Magnetic Dipole

An additional parameter, viz. the inclination of the dipole to its line of motion, makes its appearance in the dipole Cherenkov effect. However, the total loss can be obtained by summing up the two losses in the parallel and perpendicular cases. The general expression for the dipole Cherenkov losses is given by

$$ W = M_m M_r \mathcal{H}_m \mathcal{H}_r \left( \frac{H_1}{m} \right), $$

where $M$ is the dipole moment vector and $H_1$ is the magnetic field due to the magnetic monopole. Using Eq. (4) and proceeding analogously, we obtain

$$ W_\parallel = \frac{M^2_{\parallel}}{4 \pi^2 \beta^2 r^2 c^4} \int \varepsilon \left( 1 - \frac{1}{\beta^2 n^2} \right)^{3/2} \left[ \frac{2}{\tau} \left( 1 + \frac{\omega_n^2}{\omega^2} \right) \right] \cdot \omega^{5/2} d\omega, $$

$$ W_\parallel = \frac{M^2_{\parallel}}{4 \pi \beta^2 r^2 c^4} \left[ \int \varepsilon \left( 1 - \frac{1}{\beta^2 n^2} \right) \omega^3 d\omega \right] $$

$$ + \int \frac{\varepsilon}{\pi} \left( \left( 1 + \frac{\omega^2}{\omega_1^2} \right)^{3/2} - 1 \right) \omega^2 d\omega, $$

for the energy loss of a magnetic dipole moving parallel to its axis, and

$$ W_\perp = \frac{M^2_{\perp}}{12 \pi^2 c^4} \int \varepsilon^2 \mu \left( 1 - \frac{1}{\beta^2 n^2} \right)^{1/2} \frac{1}{\sqrt{2} \tau} \left( 1 + \frac{\omega_n^2}{\omega^2} \right) \cdot \omega^{5/2} d\omega, $$

$$ W_\perp = \frac{M^2_{\perp}}{8 \pi c^4} \int \varepsilon^2 \mu \left( 1 - \frac{1}{\beta^2 n^2} \right)^2 \omega^3 d\omega $$

for the energy loss of a magnetic dipole moving perpendicular to its axis in a strongly and a weakly conducting medium respectively.

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