Introduction

The band gap width is one of the most important parameters of semiconductors. Though its value, apart from rare exceptions, cannot be theoretically predicted, there exist, however, several experimental methods which yield the value with moderate accuracy. Most of these methods require relatively complicated equipments and the use of single crystals.

On the base of qualitative considerations and experimental evidences, Föchs suggested the use of diffuse reflectance spectra of powdered semiconductors for determining the band gap width and the long-wave limit of the absorption spectra of semiconductor materials. His results on powdered samples of Se, Si, Ge and CdS proved to be in good agreement with the results obtained using other methods. Hevesi compared the values of the activation energy determined on \( \text{V}_2\text{O}_5 \) single crystals using Föchs' method with diffuse reflectance measurements on powdered samples, and found satisfactory agreement. Tandon and Gupta's investigations on \( \text{Cu}_2\text{O} \) and other samples gave a less satisfactory agreement as the earlier investigations mentioned above.

The modifications of the above method, suggested in the present paper, permit to find theoretical connections between the shape of the long-wave tail of the absorption edge and the shape of the diffuse reflectance spectra. On the basis of our measurements on powdered \( \text{V}_2\text{O}_5 \) samples, it may be excepted that it will be possible to obtain further informations about the long-wave part of the absorption spectrum from its connections with the diffuse reflectance spectrum.

Theoretical Considerations

Several theories about the correlation between absorption spectra diffuse reflectance spectra of powdered solids are known. These theories (and the respective models) are apparently independent from each other, but it can be shown that there exist close connections between them.

Numerous investigations concerning the range of applicability of the different models were made, but the results published hitherto want further theoretical elaboration and experimental control.

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3. B. Lange, Die Photoelemente und ihre Anwendung, Bd. 1, Verlag Johann Ambrosius Barth, Leipzig 1940.
11. P. Kubelka, JOSA, 38, 448, 1067 [1948].
In view of the fact that most of the extensive experimental work performed was concerned with Schuster-Kubelka-Munk's theory 19, the quantitative considerations of the present paper are also founded on this theory.

Let us consider a homodisperse, isotropically scattering, not too strongly absorbing powder sample of "infinite" thickness. Then, if the particle size is sufficiently (but not excessively) great compared with the wavelength of the incident radiation, the diffuse reflectance \( R_\infty \) of the sufficiently thick ("infinite") sample can be expressed with the optical parameters of the sample as follows 19:

\[
\frac{1}{2} (1 - R_\infty)^2 = \frac{K^*}{s},
\]

where \( K^* \) is the so-called effective absorption coefficient of the sample proportional to (but not identical with) the true absorption coefficient \( K \) of the powder material, and \( s \) the so-called scattering coefficient independent from, or only slightly dependent on wavelength.

As it is well known 21, the shape of the long-wave side of the absorption spectra of solids, expressed as a function of photon energy, differs according to whether the transition determined by the symmetry of the electron wavefunctions is allowed or forbidden. For allowed transitions

\[
K_a \propto \frac{(h v - E_g)^{1/2}}{h v},
\]

whereas for forbidden transitions

\[
K_f \propto \frac{(h v - E_g)^{1/2}}{h v}.
\]

If \( K_a \) and \( K_f \) are not too great, then it follows from (1), using (2) and (3), that

\[
[f(R_\infty) h v]^2 \propto (h v - E_g),
\]

and

\[
[f(R_\infty) h v]^{1/2} \propto (h v - E_g),
\]

i.e. the intercepts of the straight lines determined by Eq. (4) in case of allowed transitions and by Eq. (5) in case of forbidden transitions will give the value of the band gap \( E_g \).

However, in the region of the long-wave tail (and especially in its initial part) the low values of \( R_\infty \) may lead to significant inaccuracies in the measurements and consequently to uncertainties of the spectral dependence of \( R_\infty \), which may influence the results of calculations. Therefore, it seems expedient to embed the semiconductor powder in a powder, colourless (without selective reflectance in the range of measurements) and not reacting chemically with the semiconductor (e.g. MgO, BaSO\(_4\), TiO\(_2\) etc.) and to measure the diffuse reflectance \( R_\infty^* \) of the "diluted" sample.

It has been shown 19 that, in this case, Eq. (1) can be written as

\[
\frac{1}{2} (1 - R_\infty^*)^2 = \frac{K^*}{s},
\]

and so relations analogous to Eqs. (4) and (5) will hold for the diluted samples (here \( c \) means the concentration of the absorbing component in the diluted sample, \( E \) its extinction coefficient at the quantum energy in question and \( s \) the scattering coefficient of the mixed sample).

Measurements and Results

For experimental checking of the relations mentioned above and the method suggested, it seemed desirable to choose as model a material the optical and semiconductor parameters of which had been determined and controlled with known accuracy. Therefore our experiments were performed with V\(_2 \)O\(_5\), studied in detail by HEVESI (for his researches on V\(_2\)O\(_5\) single crystals see 7, 22, 23).

A double beam grating spectrophotometer Type OPTICA MILANO CF 4 DR with a reflectance attachment consisting of an UBLCRicht's sphere of 15 cm diameter was used for measuring the diffuse reflectance spectra. The linear dispersion of the monochromator unit was 1.6 nm/mm, the band width of the beam issuing from the monochromator did not exceed 0.6 nm in the range of measurements. The samples were put in sample holders 3 mm deep (to be considered practically as of infinite thickness).

A MgO layer, 1 mm thick, freshly smoked on a compressed MgO layer of 2 mm thickness, was used as reference standard; its reflectance was taken as corresponding to the mean of values found in literature 24, 25, 26. BaSO\(_4\) powder of high purity was used for diluting the V\(_2\)O\(_5\) powder. The concentration of V\(_2\)O\(_5\) in the samples was 100, 10, 1 and 0.1 percent, respectively. To obtain sufficient homogeneity and

25 W. E. K. MIDDLETON and C. L. SANDERS, JOSA 41, 419 [1951].
26 P. A. TELLEX and J. R. WALDRON, JOSA 45, 19 [1955].
reproducibility, the samples were mixed in a braying mortar till the changes in reflectance on further mixing did not exceed the inherent uncertainty of measurements.

Diffuse reflectance spectra of the pure and diluted V_2O_5 samples are shown in Fig. 1.

Discussion

As can be seen from Fig. 1, with decreasing concentration the reflectance spectra are shifted towards higher reflectance, which is more favourable for the measurements; at the same time they appear somewhat reduced along the ordinates. The dilution offers the advantage of higher accuracy of measurements; on the other hand it secures the elimination of the regular reflection superposed on the diffuse reflectance spectrum, which would be particularly disturbing in the spectral ranges of higher absorption and therefore of less reflectance.

This means that the influence of the long-wave tail on the diffuse reflectance spectra in diluted samples is presumably more correctly rendered by the shape of the diffuse reflectance spectra, than in the case of pure (undiluted) samples, especially in the critical range of the absorption edge.

It is evident from the derivation of Eqs. (4) and (5) and their analogon obtained using Eq. (6) that the scattering factors s connected with the particle size of the powder sample is included in the proportionality factor omitted in the course of derivation. Therefore, in reasonable ranges of particle size and in spectral regions where the scattering coefficient is only slightly influenced by the wavelength or quantum energy, the results obtained with this method should be independent from particle size and dilution.

The functions \( \lg f(R_{\infty}) \), calculated from the results shown in Fig. 1 using the relations Eqs. (1) and (6), are plotted in Fig. 2. As can be seen, in the energy range of interest, the functions \( \lg f(R_{\infty}) \) for different dilutions can be brought into very good coincidence by parallel shifting in direction of the ordinates. This shows that in the range of absorption considered Eq. (6) holds, i.e. our basic suppositions are fulfilled. The curves of Eq. (4) for allowed direct optical transitions in case of undiluted samples, and those corresponding to the analogous equation

\[
[f(R_{\infty}^*) \ h \nu]^2 \propto (h \nu - E_g)
\]

valid for diluted samples are plotted in Fig. 3 (\( R_{\infty}^* \) is the diffuse reflectance of the diluted powder sample). The parts of the curves corresponding to higher quantum energies can be represented by
straight lines; extrapolating for zero energy, the band gap width $E_g$ can be determined. In our case the following values were obtained:

$$
E_g^{100} = 2.31 \text{ eV}, \quad E_g^{10} = 2.31 \text{ eV}, \\
E_g^1 = 2.31 \text{ eV}, \quad E_g^{0.1} = 2.32 \text{ eV}
$$

(the percentage of the $V_2O_5$ contained in the sample is denoted by the superscripts).

According to measurements of Kenny, Kannewurf and Whitemore\textsuperscript{27} and of Bodó and Hevesi\textsuperscript{7,22} on single crystals the band gap width shows slight anisotropy and is different in the directions parallel (||) with and perpendicular (⊥) to the c axis of the crystal:

Kenny et al.  
Hevesi  

$$
E_g^\parallel = 2.34 \text{ eV} \quad 2.30 \text{ eV} \\
E_g^\perp = 2.36 \text{ eV} \quad 2.32 \text{ eV}.
$$

The method suggested above evidently gives only mean values of the band gap width. A comparison of our results with those obtained by Kenny et al. and Hevesi shows very good agreement, independently from the concentration of the samples. This proves the usefulness of the method.

ed out by KENNY et al., the absorption coefficient
plotted according to both the "1/2-law" and "3/2-
law" give curves in which well defined linear sec-
tions can be found. Thus it seems difficult to decide
about the mechanism of the optical transitions on
the basis of results obtained with single crystals.
At the same time, the data for the band gap width
quoted from the papers of KENNY et al. and HEVESI
were obtained with the supposition of forbidden
direct transitions. The results of transmission mea-
surements on thin single crystals in the region of
the long-wave tail of the absorption edge are rela-
tively inexact due to the comparatively high trans-
mission values. Therefore, the true mechanism can-
not be unequivocally secured. Further investigations
seem necessary to throw more light on this contra-
dictory situation.
Studying the long-wave tail of the absorption
eedge of some solids, the absorption coefficient $K$
was found to present an exponential dependence on
energy and on inverse temperature in a certain
range of these parameters (URBACH's rule). Ac-
cording to the measurements of BODó and HEVESI
on V$_2$O$_5$ single crystals the relation

$$K = K_0 \exp(\beta h \nu/k T)$$

holds, where $K_0$ means a constant, $k$ is BOLTZMANN's
constant, $T$ the absolute temperature and $\beta$ a con-
stant about 1/2. Accordingly, it is to be expected
that, on the basis of Eqs. (1) and (6), in a certain
spectral range in the region of the long-wave tail,
the following two relations will be fulfilled:

$$\lg f(R_\infty) \propto \beta h \nu/k T,$$  (10)

$$\lg f(R_\infty') \propto \beta h \nu/k T,$$  (11)

i.e. $\lg f(R_\infty)$ should be a linear function of the
quantum energy $h \nu$. As can be seen in Fig. 2, this
linearity is well fulfilled in the quantum energy
range from 2.2 eV to 2.34 eV; this confirms the
validity of URBACH's rule (from the energetic point
of view). From Eqs. (10) and (11) also the value
of parameters $\beta$ can be determined, namely

$$\beta = k T \frac{\Delta \lg f(R_\infty)}{\Delta (h \nu)} = k T \frac{\Delta \lg f(R_\infty')}{\Delta (h \nu)}.$$  (12)

For the slope of the linear section a value of
about 0.24 was obtained, independently from con-
centration, against 0.47 - 0.52 given for single
crystals. For the time being there are not
sufficient experimental and theoretical results to
understand this deviation. We should like to point
at our earlier experimental results according to
which in case of powder samples consisting of par-
ticles of small optical thickness, the function $f(R_\infty)$
is not a linear function of the quotient $(K*/S)$, as
supposed in Schuster-Kubelka-Munk's theory.
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