The First Order Density Corrections Including Ternary Collisions in a Boltzmann-Landau Gas

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The first order density corrections to the coefficients of viscosity ($\eta$) and thermal conductivity ($\kappa$) in a Boltzmann-Landau gas have been calculated including the effect of ternary collisions.

1. Introduction

The aim of this paper is to give a method of formal expansions in powers of density of the coefficients of thermal conductivity ($\kappa$) and viscosity ($\eta$) of a moderately dense Boltzmann-Landau gas, based on the assumptions given in a previous paper. We shall consider here particularly the contributions of the ternary collision process to the transport integral equations. This will describe completely the first order density correction to the transport coefficients (the quadruple process does not contribute to the first order density correction). The general integral equation governing these transport phenomena in the first order Chapman-Enskog approximation in terms of the quadruple process does not contribute to the first order density correction. The general integral equation including multicollision processes will be assumed as given by

$$L(X) = \int f_0 f_0^P (p) f_0 (p_1) [X(p_1') + X(p')] - X(p_1) - X(p) \, dp_1 \, dp \, dp'$$

subject to the subsidiary condition [(2.7a), 1]. First of all, we shall see that the pressure tensor $\pi_{\mu \nu}$ and heat flow $Q$ can be expressed, in this first order Chapman-Enskog approximation in terms of $X(p)$ instead of $\varphi$ in ref. 1. This might seem useful, since (1.1) is a symmetric integral equation. The general expressions of $\pi_{\mu \nu}$ and $Q$ will not be deduced here. Instead we shall use the known results given earlier by Grossmann.

2 A. The Pressure Tensor $\Pi_{\mu \nu} (\mu \neq \nu)$

The pressure tensor for any arbitrary distribution $f$ is given in 2-4 to be

$$\pi_{\mu \nu} = \int (P_{\mu} - m u_{\mu}) \left( \frac{2 e_p (f)}{P^2} - u_{\nu} \right) f(P) \, dP \quad (2.1)$$

for $\mu \neq \nu$.

Here $u = \langle P/m \rangle$ is interpreted as the mean velocity. For the first order Chapman-Enskog approximation we have demanded (as our subsidiary condition):

$$\langle P/m \rangle = \langle P/m \rangle_0,$$

where $f = f_0 + f_1$, $f_1 \ll f_0$.

According to 1, (2.2c) we can put in the first order in $f_1$,

$$e_p (f) = e_p (f_0) + \int \tilde{F} (P, Q; f_0) f_1 (Q) \, dQ \quad (2.2)$$

Since, for $f = f_0$, the non-diagonal elements of $\pi_{\mu \nu}$ vanish, we have by (2.1) and (2.2), in the first order in $f_1$:

$$\pi_{\mu \nu} = \int p_{\mu} \left( \frac{2 e_p (f_0)}{P^2} - u_{\nu} \right) f_1 (P) \, dP \quad (2.3)$$

$$+ \int p_{\nu} \left( \frac{2 e_p (f_0)}{P^2} - u_{\mu} \right) F (P, Q; f_0) f_1 (Q) \, dP \, dQ$$

where $P = P - m u$.

(2.3) is similar to that given by Grossmann with the exception that $F(P, Q)$ in 4 is replaced by $\tilde{F} (P, Q; f_0)$ to include multicollision processes. In order to express (2.3) in terms of $X(P)$, we recall,

3 S. Grossmann, Nuovo Cimento 37, G 98 [1964].
Then with $f_i = f_0 \varphi$, (2.3) becomes

$$
\tau_{\mu\nu} = - \frac{1}{\beta} \int P_{\mu} \frac{3f_0}{3P_{\nu}} \varphi(P) \, dP
+ \int P_{\mu} \frac{3}{3P_{\nu}} \tilde{F}(P, Q; f_0) f_1(Q) f_0(P) \, dP \, dQ.
$$

Integrating the second term partially w.r.t. to $P_{\nu}$ and collective terms, we get

$$
\tau_{\mu\nu} = - \frac{1}{\beta} \int P_{\mu} \frac{3f_0}{3P_{\nu}} X(P) \, dP
$$

where we have used

$$
X(P) = \varphi(P) + \beta \int \tilde{F}(P, Q; f_0) f_1(Q) \, dQ
$$

(2.5')

according to $^1$, (2.2b). Now under the restrictions on $F_r(P_1, \ldots, P_v)$ used in ref. $^1$, it is clear from (1.1) that $X(P)$ depends on $P$ only through $p = P - mu$. Also using (2.4) we find

$$
\frac{m}{\beta} \frac{3f_0}{3P_{\nu}} = - p_{\nu} \left( 1 + \frac{m}{p} \frac{3\tilde{F}(p; f_0)}{3p} \right) f_0(p)
$$

$$
= - S_1(p) f_0(p) \, p_{\nu}.
$$

Thus the pressure tensor is, for $\mu = \nu$,

$$
\tau_{\mu\mu} = \frac{1}{m} \int P_{\mu} P_{\nu} S_1(p) f_0(p) X(p) \, dP
$$

(2.7)

2B. The Heat Flow $Q$

In the general case of arbitrary $f$, the heat flow $Q$ is defined by $^2$:

$$
Q_i = \int \varepsilon_{\mu}(f) \frac{3\varepsilon_{\mu}(f)}{3p_{\nu}} f_0(p) \, dp
$$

(2.8)

where we are considering the physical situation $u = 0$ substituting (2.2) and $f = f_0 + f_1$, $f_1 = f_0 \varphi$ in (2.8), $Q$ can be put in the following form (in the first order in $f_1$)

$$
Q_i = \int \varepsilon_{\mu}(f_0) \frac{3\varepsilon_{\mu}(f_0)}{3p_{\nu}} f_0(p) \varphi(p) \, dp
$$

$$
- \int \varepsilon_{\mu}(f_0) \tilde{F}(p, q; f_0) \frac{3f_0}{3p_{\nu}} f_1(q) \, dp \, dq.
$$

(2.9)

Also, since in this case

$$
\frac{1}{f_0} \frac{3f_0}{3p_{\nu}} = - \beta \frac{3\varepsilon_{\mu}(f_0)}{3p_{\nu}},
$$

it turns out that

$$
Q_i = - \frac{1}{\beta} \int \varepsilon_{\mu}(f_0) S_1(p) f_0(p) X(p) \, dp
$$

(2.10)

where we have used (2.5') to obtain (2.10).

Finally, using (2.6), we have

$$
Q_i = \frac{1}{m} \int p_{\nu} \varepsilon_{\mu}(f_0) S_1(p) f_0(p) X(p) \, dp
$$

(2.11)

The results (2.7) and (2.11) include multi-collision processes in the Boltzmann-Landau gas and take the form discussed in Ref. $^6$ in case of the binary collision approximation. To find $\tau_{\mu\nu}$ and $Q$ and therefore the coefficients $\eta$ and $\lambda$ of viscosity and thermal conductivity respectively, we need to find the formal solution for $X(P)$ in the symmetric linear integral equation, which we shall now carry out.

3. The Formal Solution of the Transport Equation

Equation (1.1) looks similar to the ordinary Chapman-Enskog equation for a Boltzmann gas. The solutions of the homogeneous equation are, in our case, the five collision invariants $\psi_i = 1, p_{\mu}$ and $\varepsilon_{\mu}(f_0)$. Therefore, using the usual procedure, we can write down the general solution of (1.1) in the following form:

$$
X(p) = - \left[ \frac{1}{T} \frac{3T}{3x_{\mu}} p_{\mu} A(p) + A_{ij} B_{ij}(p) + a + b_{\mu} p_{\mu} + c \varepsilon_{\mu}(f_0) \right]
$$

(3.1)

where $A_i$ and $B_{ij}$ satisfy the integral equations:

$$
L(A_i) = f_0(p) p_{\mu} R(p), \quad \text{with} \quad A_i = p_{\mu} A(p),
$$

(3.2)

$$
L(B_{ij}) = f_0(p) S_{ij}(p), \quad \text{with} \quad B_{ij} = \frac{\beta}{m} (p_{\mu} p_{\mu} - \frac{1}{3} p^2 \delta_{ij}) B_i(p) + \delta_{ij} B_2(p).
$$

(3.3)

The Eqs. (2.3) and (3.3) are clearly solvable, since

$$
\int \varphi_i(p) p_{\mu} R(p) f_0(p) \, dp = \int \varphi_i(p) S_{ij}(p) f_0(p) \, dp = 0.
$$

(3.4)

We have now to determine the five constants $a$, $b$, $c$ in (3.1). This we shall do by using the subsidiary conditions \(^1\), (2.7a):
\[
    \int f_0(p) X(p) \varphi_i(p) \, dp = 0
\]
(3.5a) where $\varphi_i(p) = R^{-1} \psi_i(p)$ (3.5b)

Now since, according to \(^1\), (2.5)
\[
    \varphi_i(p) = \psi_i(p) + \sum_{s=1}^{\infty} (-\beta)^s \int \tilde{F}(p, q, f_0) \ldots \tilde{F}(q_{s-1}, q_s, f_0) \psi_i(q_s) \prod_{i=1}^{s} (f(q) \, dq)
\]
(3.5c)

it is easy to see that: for $\varphi_i = 1$, $\varphi_i(p) = \psi_i(p)$, for $\varphi_i = \varepsilon_p$, $\varphi_i(p) = p_i \varphi_2(p)$, and for $\varphi_i = \varepsilon_p$, $\varphi_3(p) = \varphi_3(p)$. Then, as in ref. \(^6\), we can use the subsidiary conditions to obtain

\[
    \int f_0(p) A_i \delta_{ij} B_2 + a + c \varepsilon_p(f_0) \varphi_3(p) \, dp = 0 .
\]
(3.6a)

and

\[
    \int f_0(p) A_i \delta_{ij} B_2 + a + c \varepsilon_p(f_0) \varphi_3(p) \, dp = 0 .
\]
(3.6b)

Here we have used $\int (p_i p_j - \frac{1}{2} p^2 \delta_{ij}) G(p) \, dp = 0$, where $G$ is a function of $p$ alone. As usual \(^6\), (3.6b) shows that $c_i \sim (1/T) \delta T/\delta x_i$ and the term $b_p A_p$ can be absorbed in the first term in (3.1) and $b_i = 0$, so that (3.6b) yields the subsidiary condition

\[
    \int A(p) \varphi_2(p) f_0(p) p^2 \, dp = 0 .
\]
(3.8a)

Similarly, (3.6a) and (3.6c) show that $a$ and $c$ are proportional to $A_i \delta_{ij}$ and therefore the terms involving $a$ and $c$ can be absorbed in the second term of (3.1) by redefining $B_2$ which thus satisfies the subsidiary conditions:

\[
    \int f_0(p) B_2(p) \varphi_i(p) \, dp = \int f_0(p) B_2(p) \varphi_3(p) \, dp = 0 .
\]
(3.8b)

(3.8a, b) are the new versions of the subsidiary conditions (2.10a, b) of Ref. \(^6\). Thus

\[
    X(p) = \left[ 1 + \frac{1}{T} \frac{p_i A(p) + A_i B_{ij}(p)}{\varepsilon_p} \right]
\]
(3.9)

where $A_i$ and $B_{ij}$ satisfy the symmetric linear integral Eqs. (3.2) and (3.3) satisfying the subsidiary conditions (2.10a, b) of Ref. \(^6\).

4. The Effect of Ternary Collision on the First Order Density Corrections to Transport Coefficients

The first density corrections to $\eta$ and $\lambda$ already obtained in the binary collision approximation \(^6\) will be extended here to include the ternary collision processes. However the previous calculations of \(^3\) and \(^6\) will be used whenever necessary in order to avoid repetitions.

(A) Coefficient of viscosity ($\eta$)

From the non-diagonal elements of the pressure tensor $\tau_{\mu\nu}$ given by (2.7) and from (3.9) we have

\[
    \tau_{\mu\nu} = - \frac{1}{m} \frac{\delta T}{\delta x_i} \int p_i p_j S_1(p) f_0(p) p_i A(p) \, dp
\]

(4.1)

The usual argument \(^6\) shows that the term involving $\delta T/\delta x_i$ in (4.1) vanishes. Thus, for $\mu = \nu$,

\[
    \tau_{\mu\nu} = - \frac{A_i}{m} \int p_i p_j S_1(p) f_0(p) \left[ \frac{1}{m^2} (p_i p_j - \frac{1}{2} p^2 \delta_{ij}) B_1(p) \, dp + \delta_{ij} B_2(p) \right].
\]

(4.2)

It is clear that, in the sum over $i$ and $j$, the r.h.s. of (4.2) will vanish unless $i = \mu$, $j = \nu$, so that $\tau_{\mu\nu} = - 2 \eta A_i \eta$, where the coefficient of viscosity $\eta$ is given by

\[
    \eta = \frac{\beta}{2 m^2} \int p_i^2 p_j^2 S_1(p) f_0(p) B_1(p) \, dp
\]

(4.3)

where we have used

\[
    \int G(p) p_i^2 p_j^2 \, dp = \frac{1}{15} \int G(p) p^4 \, dp ,
\]

for any scalar function $G(p)$. (4.3) looks much simpler than given earlier \(^6\). Of course the $B_{ij}$ here satisfy essentially a different integral equation (a symmetric one) and are subjected to subsidiary conditions (3.8b) different from those obtained earlier \(^6\). We now proceed to find the first order density correction to $\eta$ directly from (4.3). As before \(^6\), we take

\[
    B_1(p) = B_1^\eta(p) + n B_1^\lambda(p) + \ldots .
\]
(4.4)
Also

\[ f_0(q)/f_0(g) \, dq = \omega_0(q) [1 + n \Theta(q)] \quad (4.5a) \]

where

\[ \Theta(q) = 2[B(T) - F_{2,o}(p)] \quad (4.5b) \]

Defining \( F_0(p) = F_{2,o}(p) \), we have

\[ F_0(p) = \int f_2(p, q) \, \omega_0(q) \, dq . \quad (4.5c) \]

(hereafter the index “0” will correspond to \( n = 0 \)).

Here we notice that in (4.5a), \( F_3(p_1, p_2, p_3) \) does not contribute to the first order density correction to \( f_0 \). Now with

\[ S_1(p) = 1 + n S_1^1, \quad S_1^1 = \frac{m}{p} \frac{\partial F_0}{\partial p} \]

(see ref. 6) we have from (4.3):

\[ \eta = \eta_0 + n \eta_1 , \quad (4.6a) \]

where

\[ \eta_0 = \frac{\beta}{30 m^2} \int p^4 B_1^0(p) \, \omega_0(p) \, dp \quad (4.6b) \]

\[ \eta_1 = \frac{\partial G}{\partial n} \quad \xi \quad \left[ \frac{\beta}{30 m^2} \int p^4 [B_1^1(p) + S_1^1(p) B_1^0(p) + B_1^0(p) \Theta(p)] \, \omega_0(p) \, dp . \quad (4.6c) \]

The contribution to \( \eta_1 \) of the ternary collision appears through \( B_1^1(p) \), which in turn is determined from an integral equation containing triple collisions, as we shall see now from the density expansion of (3.3), where \( L(X) \) can be written as, according to 1, (3.8) and 6, (3.9):

\[ L(X) = -X(p) f_0(p) \sum(p) + \int K(p, q) X(q) \, dq = D f_0 , \quad (4.7a) \]

\[ K'(p, q; f_0) = \int [2 \sigma(s q | r p; f_0) f_0(s) - \sigma(s t | q p; f_0) f_0(p)] \, ds \, dt , \quad (4.7b) \]

\[ \sum(p) = \int \sigma(s t | r p; f_0) f_0(r) \, ds \, dt \, dr = \int G(r, p; f_0) f_0(r) \, dr \quad (4.7c) \]

where

\[ G(r, p) = \int \sigma(s t | r p; f_0) f_0(r) \, ds \, dt \quad (4.7d) \]

Now assuming \( X = (A_0 + n X_1 + ...) \), we obtain from (4.7c) and (4.12a), to the first order in the

Also

\[ f_0(q)/f_0(g) \, dq = \omega_0(q) [1 + n \Theta(q)] \quad (4.5a) \]

where

\[ \Theta(q) = 2[B(T) - F_{2,o}(p)] \quad (4.5b) \]

Defining \( F_0(p) = F_{2,o}(p) \), we have

\[ F_0(p) = \int f_2(p, q) \, \omega_0(q) \, dq . \quad (4.5c) \]

(hereafter the index “0” will correspond to \( n = 0 \)).

Here we notice that in (4.5a), \( F_3(p_1, p_2, p_3) \) does not contribute to the first order density correction to \( f_0 \). Now with

\[ S_1(p) = 1 + n S_1^1, \quad S_1^1 = \frac{m}{p} \frac{\partial F_0}{\partial p} \]

(see ref. 6) we have from (4.3):

\[ \eta = \eta_0 + n \eta_1 , \quad (4.6a) \]

where

\[ \eta_0 = \frac{\beta}{30 m^2} \int p^4 B_1^0(p) \, \omega_0(p) \, dp \quad (4.6b) \]

\[ \eta_1 = \frac{\partial G}{\partial n} \quad \xi \quad \left[ \frac{\beta}{30 m^2} \int p^4 [B_1^1(p) + S_1^1(p) B_1^0(p) + B_1^0(p) \Theta(p)] \, \omega_0(p) \, dp . \quad (4.6c) \]

The contribution to \( \eta_1 \) of the ternary collision appears through \( B_1^1(p) \), which in turn is determined from an integral equation containing triple collisions, as we shall see now from the density expansion of (3.3), where \( L(X) \) can be written as, according to 1, (3.8) and 6, (3.9):

\[ L(X) = -X(p) f_0(p) \sum(p) + \int K(p, q) X(q) \, dq = D f_0 , \quad (4.7a) \]

\[ K'(p, q; f_0) = \int [2 \sigma(s q | r p; f_0) f_0(s) - \sigma(s t | q p; f_0) f_0(p)] \, ds \, dt , \quad (4.7b) \]

\[ \sum(p) = \int \sigma(s t | r p; f_0) f_0(r) \, ds \, dt \, dr = \int G(r, p; f_0) f_0(r) \, dr \quad (4.7c) \]

where

\[ G(r, p) = \int \sigma(s t | r p; f_0) f_0(r) \, ds \, dt \quad (4.7d) \]

Now assuming \( X = (A_0 + n X_1 + ...) \), we obtain from (4.7c) and (4.12a), to the first order in the
density \( n \):
\[
\int K(p,q) \, X(q) \, dq = n \int \omega_0(q) \, K_0(p,q) \, X_0(q) \, dq \\
+ n^2 \int \omega_0(q) \{ K_0(p,q) \, X_1(q) + X_0(q) \left[ K_1(p,q) + K_0(p,q) \, \Theta(q) \right] \} \, dq.
\]
(4.13)

The right hand side of (4.7a), namely \( D /_0 \), has been calculated in [1]. It is however easy to verify that, in the first order in density \( n \), \( D /_0 \) remains the same as given in section 3 of ref. [6]. Putting \( \Theta = B_{ij}, \) i.e., \( \Theta_0 = B_{ij}^0, \) \( \Theta_x = B_{ij}^x \), we obtain

\[
I(B_{ij}) = - \frac{\beta}{m} \omega_0(p) (p_i p_j - \frac{1}{3} \rho^2 \delta_{ij})
\]
(4.14a)

and following the same procedure as in case (A), we get, \( \lambda = \lambda_0 + n \lambda_1 \),

where

\[
\lambda_0 = \int \frac{p^4}{2m} \, A^0(p) \, \omega_0(p) \, dp,
\]
(4.19b)

\[
\lambda_1 = (\frac{d\lambda}{dn})_{n=0}
\]

\[
= \int \left[ \frac{p^2}{2m} S_1(p) + \frac{p^2}{2m} \Theta(p) \right] A^0(p)
\]
+ \frac{p^2}{2m} A^1(p) \, \omega_0(p) \, dp.
\]
(4.19c)

As in case (A), the integral equation for \( A_0^1 \) and \( A_1^1 \) can be calculated using (4.13) and

\[
A_1^2 = p \, A_2^1(p) \, (x = 0, 1).
\]

The results [putting \( X = p \, A(p) \) in (4.13)] are:

\[
I(A_0^1) = - \omega_0(p) \left( \frac{\beta p^2}{2m} - \frac{5}{2} \right) p_i,
\]
(4.20a)

\[
I(A_1^1) = - \omega_0(p) \, \partial(p) \, p_i
\]
(4.20b)

with

\[
\omega_0(p) \, \partial(p) \, p_i = \omega_0(p) \left[ \partial^{(b)}(p) + \partial^{(c)}(p) \right] p_i
\]
(4.20c)

where \( \Theta^{(b)}(p) \) and \( \Theta^{(c)}(p) \) are contributions from the binary [6] and ternary collision processes, as in case (A):

\[
\omega_0(p) \, \partial^{(c)}(p) \, p_i = \int \omega_0(q) \, q_i \, A^0(q) \, K_1^{(c)}(q,p,q) \, dq.
\]
(4.20d)

The subsidiary conditions in each order of density corrections can also be found by putting \( \psi_{\mu} = p_{\mu} \) in (3.5c), so that

\[
p_{\mu} \, \varphi_2(p) = p_{\mu} - n \, \beta \int q \, F_2(p,q) \, \omega_0(q) \, dq.
\]
(4.21a)

Multiplying throughout by \( p_{\mu} \) and adding we get

\[
\varphi_2(p) = 1 - n \, \varphi_1^1(p)
\]
(4.21b)

where

\[
\varphi_1^1(p) = \frac{\beta}{p^2} (p \cdot q) \, F_2(p,q) \, \omega_0(q) \, dq.
\]
(4.21c)
The subsidiary condition (3.8a) then takes the form:
\[ fA*(p) \omega_0(p) p^2 dp = 0 \] (4.22a)
\[ fA^p) \omega_0(p) p^2 dp \] (4.22b)
Similar conditions can also be found for \( B_2^9 \) and \( B_2^1 \) from (3.8b).

5. Discussion

In the previous section we have considered the effect of triple collision only. The calculations for the higher collision processes and therefore the density expansion of the transport coefficients can be carried out similarly. In the above calculation we see that \( F^S_p(P, Q; f_0) \) does not appear explicitly, the whole contribution to the first order density correction appears through \( \sigma_1(K) \) [and equivalently \( K_1^{(b)}(p, q) \)]. \( F^S_p \) will appear explicitly in the second density correction. The higher corrections do not seem to bring any better insight into the theory for the present and so we have omitted them here.

Berichtigung:


Page 117. Eq. (2.2a) read:
\[ L(X) = \int \sigma(J_0) J_0(P) J_0(P_1) \left[ x(P_1') + x(P') - x(P_1) - x(P) \right] dP' dP_1 dP' = Df_0, \] (2.2a)
Eq. (2.5) read:
\[ \varphi(P) = R^{-1} X(P) = X(P) + \sum_{s=1}^{\infty} (-\beta)^s \int \tilde{F}(P, Q_1; f_0) \ldots \tilde{F}(Q_{s-1}, Q_s; f_0) X(Q_s) \prod_{i=1}^{s} (f_0(Q_i) dQ_i). \] (2.5)
Eq. (2.6a) read:
\[ + \sum_{s} (-\beta)^s \int \tilde{F}(P, Q_1; f_0) \ldots \tilde{F}(Q_{s-1}, Q_s; f_0) X(Q_s) \varphi(P) J_0(P) \prod_{i=1}^{s} (f_0(Q_i) dQ_i) dP. \] (2.6a)
Eq. (2.6b) read:
\[ A = \int f_0(P) \varphi(P) X(P) dP + \sum_{s} (-\beta)^s \int \tilde{F}(Q_s, Q_1; f_0) \ldots \tilde{F}(Q_{s-1}, P; f_0) \varphi(Q_s) J_0(P) X(P) \prod_{i=1}^{s} (f_0(Q_i) dQ_i) dP \]
\[ = \int f_0(P) \varphi(P) X(P) dP + \sum_{s} (-\beta)^s \int \tilde{F}(Q_s, Q_{s-1}; f_0) \ldots \tilde{F}(Q_1, P; f_0) \varphi(Q_s) J_0(P) X(P) \prod_{i=1}^{s} (f_0(Q_i) dQ_i) dP \]
\[ = \int f_0(P) X(P) dP \{ \varphi(P) + \sum_{s} (-\beta)^s \int \tilde{F}(P, Q_1; f_0) \ldots \tilde{F}(Q_{s-1}, Q_s; f_0) \varphi(Q_s) \prod_{i=1}^{s} (f_0(Q_i) dQ_i) \} \] (2.6b)

Page 120, Eq. (3.13b) read:
\[ S_1(p) = \left( 1 + \frac{n m}{p} \frac{\partial \tilde{F}(p, f_0)}{\partial p} \right). \] (3.13b)


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Page 125. Eq. (4.17b) read:
\[ \lambda = \frac{1}{m T} \int p_n p_v \epsilon_p(J_0) S_1(p) f_0(p) A(p) dP. \] (4.17b)
Page 125 under Eq. (4.20c) read: \( \vartheta^{(b)}(p) \) and \( \vartheta^{(a)}(p) \).