Soft-Core Model in Nuclear Matter Calculations

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Using a Thomas-Fermi method developed by Kumar, Le Couteur and Roy 1, it is shown here that the two-body soft-core potential suggested by Köhler and Wagemare 2 does not give rise to correct binding energy and equilibrium density in nuclear matter calculations.

I. Introduction

The use of Thomas-Fermi method for calculating nuclear properties has received an impetus by the recent works of Bethe 3 and his cowokers. It is well known that by its very nature the Thomas-Fermi method gives only the overal properties, such as the semi-empirical mass formula, yet the simplicity in a Thomas-Fermi calculation makes it worthwhile, particularly since it can be used as the starting ground for more elaborate Hartree-Fock calculations.

Some time ago, Kumar, Le Couteur and Roy 1 had obtained a Thomas-Fermi method from the K-matrix theory of Brueckner where they had given a simple method for testing the merits of a two-body nuclear potential. It seems natural to us that the first thing one can do with a new two-body potential is to apply the above test to it. If it is found satisfactory, then one can go in for further calculations for nuclear matter and for finite nuclei using either the Thomas-Fermi or the Hartree-Fock method.

As our first choice we have taken the soft-core potential of Köhler and Wagemare 2. We may also mention that at present we are working on the Reid 4 potentials.

II. Calculations and Results

Kumar, Le Couteur and Roy 1 derived an expression for the energy density for a nucleus which, with equal number of neutrons and protons and omitting the Coulomb potential energy, takes the form

\[ E = c \rho^{2/3} - a_1 \rho^{2/3} + \frac{2^{1/3}}{3} \tau_0 a_2 \rho^{2/3} + a_3 (\nabla \rho)^2 \]  

(1)

where \( c = 3.6 \hbar^2 / 2M \) and \( \tau_0 = \frac{3}{2} (3/\pi)^{1/3} (2\pi)^3 \), \((M = \text{nucleon mass})\), and \( a_1 \) and \( a_2 \) are the first and second moments of Brueckner K-matrix with \( a_3 \) given through \( a_2 \) in a rather complicated manner. They showed that knowing \( a_1 \) and \( a_2 \) one could find \( \rho_0 \), the equilibrium density, and \( \lambda \), the binding energy per particle for nuclear matter, by using the Hugenholtz and Van Hove 5 condition that the binding energy per particle in nuclear matter be minimum, i.e.,

\[ \frac{d}{d \rho_0} (\epsilon_{nm}) = 0, \]  

(2)

where \( \epsilon_{nm} \) is the energy density for nuclear matter.

It is easily seen that one can test the merits of a nuclear two-body potential by comparing the values found from the above type of calculations with the presently accepted values 6, namely,

\[ \rho_0 = 0.17 \text{ fm}^{-3} \]  

and \( \lambda = 16 \text{ MeV} \).

This is what we propose to do with Köhler and Wagemare's 2 form of a two-body potential

\[ v(r) = v_0 \left( \frac{r^n}{r_0^n} \right) \exp\left( -r^2/r_0^2 \right), \]  

(3)

where \( v_0 \) represents the strength of the potential, \( c \) is the core radius and \( n \) determines the "hardness" of the core.

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3 N. M. Hugenholtz and Van Hove, Physica 24, 263 [1958].
The parameters $a_1$ and $a_2$ in (1) are given as:

$$a_1 = -\frac{3}{16} \int K(r, r') \, dr \, dr', \tag{4}$$

$$a_2 = -\frac{3q}{128} \int K(r, r') \, r^2 \, dr \, dr' \tag{5}$$

with $q = 0.6$ and $K(r, r')$ is the Brueckner reaction matrix.

We use the separation method of MOSKOWSKI and SCOTT\(^7\) in which the interaction $v$ is divided into a short-range and a long-range part $v = v_s + v_l$ at a separation distance $d$. With suitable choice of $d$ we can write the reaction matrix as follows:

$$K = v_l + (\Omega_s - D) \, e((Q - 1)(\Omega_s - 1)$$

$$+ (\Omega_s - 1)(e_0 - e)(\Omega_s - 1)$$

$$+ \text{higher order terms}\tag{6}$$

From (11) and (5) we get

$$a_2 = -\frac{3}{16} K(0, 0). \tag{9}$$

Also, differentiating Eq. (7) with respect to $p$ twice we have

$$\nabla^2 K(p, p') = -\int_{-\infty}^{+\infty} K(r, r') \, r^2 \, dr \, dr'. \tag{10}$$

or

$$\nabla K(p, p') \Big|_{p_0=0} = -\int_{-\infty}^{+\infty} K(r, r') \, r^2 \, dr \, dr'. \tag{11}$$

From (11) and (5)

$$a_2 = \frac{3q}{128} \nabla^2 K(p, p') \Big|_{p_0=0} \tag{12}$$

According to MOSKOWSKI and SCOTT\(^7\) the long-range part of the two-body potential is given as:

$$v(p, p') = \sum_l (2l + 1) \, v_l(p, p') \tag{13}$$

with

$$v_l(p, p') = 4\pi \int_0^\infty \left( j_l(p \cdot r) \, v(r) \, j_l(p' \cdot r) \right) \, r^2 \, dr$$

where $j_l(p \cdot r)$ denotes the spherical Bessel function.

We know that $j_l(0)$ is zero except for $l=0$. So we need consider the case $l=0$ only.

d e\pm m\over d\phi} = \epsilon_{nm} = \lambda (17)

where $\lambda$ is the binding energy per particle of the nuclear matter. By drawing graphs of $\epsilon_{nm}/Q$ and $d\epsilon_{nm}/d\phi$ against $Q$, for integral values of $n$ from 1 to 8 in the two-body potential (3), we find that these graphs intersect for values of $\rho_0$ ranging between 1.55 and 1.65 and of $\lambda$ between 70 and 100 which are much too high to be reasonable values. This probably means that the two-body potential of Köhler and Wagmare is too soft.

If we now consider the contributions to $a_1$ and $a_2$ due to the Pauli and dispersion terms in the reaction matrix, we see that these contributions do not improve the situation. The contributions due to the Pauli terms are very small, and the contributions due to the dispersion term do not improve the situation very much. In fact, it is found that to get the correct values of $\rho_0 = 0.17$ and $\lambda = 16$ MeV we have to increase the value of $a_2$ by 3 times $a_2$. Hence it is very unlikely that one will get correct results by adding contributions from the Pauli and dispersion terms to the main term $(a_2)_L$, the long-range part of the interaction. Since the soft-core two-body potential used here does not give reasonable values for the equilibrium density ($\rho_0$) and the binding energy per particle in nuclear matter ($\lambda$), we think it unlikely to be of much use for further calculations for nuclear matter or finite nuclei.

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