Soft-Core Model in Nuclear Matter Calculations

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Using a Thomas-Fermi method developed by KUMAR, LE COUTEUR and ROY, it is shown here that the two-body soft-core potential suggested by KÖHLER and WAGMARE does not give rise to correct binding energy and equilibrium density in nuclear matter calculations.

I. Introduction

The use of Thomas-Fermi method for calculating nuclear properties has received an impetus by the recent works of BETHE and his coworkers. It is well known that by its very nature the Thomas-Fermi method gives only the overall properties, such as the semi-empirical mass formula, yet the simplicity in a Thomas-Fermi calculation makes it worthwhile, particularly since it can be used as the starting ground for more elaborate Hartree-Fock calculations.

Some time ago, KUMAR, LE COUTEUR and ROY had obtained a Thomas-Fermi method from the K-matrix theory of Brueckner where they had given a simple method for testing the merits of a two-body nuclear potential. It seems natural to us that the first thing one can do with a new two-body potential is to apply the above test to it. If it is found satisfactory, then one can go in for further calculations for nuclear matter and for finite nuclei using either the Thomas-Fermi or the Hartree-Fock method.

As our first choice we have taken the soft-core potential of KÖHLER and WAGMARE. We may also mention that at present we are working on the REID potentials.

II. Calculations and Results

KUMAR, LE COUTEUR and ROY derived an expression for the energy density for a nucleus which, with equal number of neutrons and protons and omitting the Coulomb potential energy, takes the form

$$\mathcal{E} = c q^{\lambda_0} - a_1 q^{\lambda_2} + \frac{2^{\lambda_1}}{3} \tau_0 a_2 q^{\lambda_3} + a_3 (\nabla q)^2$$  \(1\)

where \(c = 3.6 \hbar^2/2M\) and \(\tau_0 = \frac{3}{2} (3/8 \pi)^{1/3} (2/\pi)^2\), \((M = \text{nucleon mass})\), and \(a_1\) and \(a_2\) are the first and second moment of Brueckner K-matrix with \(a_3\) given through \(a_3\) in a rather complicated manner.

They showed that knowing \(a_1\) and \(a_2\) one could find \(q_0\), the equilibrium density, and \(\lambda\), the binding energy per particle for nuclear matter, by using the HUGENHOLTZ and VAN HOVE condition that the binding energy per particle in nuclear matter be minimum, i.e.,

$$\frac{d}{dq} (\varepsilon_{nm} q) = 0,$$  \(2\)

where \(\varepsilon_{nm}\) is the energy density for nuclear matter. It is easily seen that one can test the merits of a nuclear two-body potential by comparing the values found from the above type of calculations with the presently accepted values, namely,

\(q_0 = 0.17 \text{ fm}^{-3}\) and \(\lambda = 16 \text{ MeV}\).

This is what we propose to do with KÖHLER and WAGMARE’s form of a two-body potential

$$v(r) = v_0 \left( \frac{r^n - c^n}{r^n} \right) \exp (-r^2/r_0^2),$$  \(3\)

where \(v_0\) represents the strength of the potential, \(c\) is the core radius and \(n\) determines the “hardness” of the core.

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The parameters $a_1$ and $a_2$ in (1) are given as

$$a_1 = -\frac{3}{16} \int K(r, r') \, dr \, dr',$$

(4)

$$a_2 = -\frac{3q}{128} \int K(r, r') \, r^2 \, dr \, dr'$$

(5)

with $q = 0.6$ and $K(r, r')$ is the Brueckner reaction matrix.

We use the separation method of MOSKOWSKI and SCOTT in which the interaction $v$ is divided into a short-range and a long-range part

$$v = v_s + v_l$$

at a separation distance $d$. With suitable choice of $d$ we can write the reaction matrix as follows

$$K = v_l + (Q_s - D) \, e(Q_l - 1) + (Q_t - 1) \, e(Q_t - 1) + \text{higher order terms}$$

(6)

From (11) and (5),

$$a_2 = -\int K(r, r') \, dr \, dr'$$

(7)

Hence,

$$a_1 = -\frac{3}{16} K(0, 0).$$

(9)

Also, differentiating Eq. (7) with respect to $p$ twice we have

$$\nabla^2 K(p, p') = -\int K(r, r') \, r^2 \, dr \, dr'$$

(10)

or

$$\left. \nabla^2 K(p, p') \right|_{p=0} = -\int K(r, r') \, r^2 \, dr \, dr'$$

(11)

We first consider the long-range term $v_l$ of the reaction matrix. From Eqs. (9) and (14), we have

$$(a_1)_L = -\frac{3}{16} v_s(0, 0) = -\frac{3 \pi}{4} \int v(r) \, r^2 \, dr$$

(15)

where $(a_1)_L$ denotes the contribution to $a_1$ from the long-range term alone. $(a_2)_L$, the contribution to $a_2$ from the long-range term, is easily found to be

$$(a_2)_L = -\frac{3q}{32} \int v(r) \, r^2 \, dr = \frac{q}{8} (a_1)_L$$

(16)

using Eq. (12).

For simplicity of calculation we first consider $(a_1)_L$ and $(a_2)_L$, the main contributory terms to $a_1$ and $a_2$. From the Hugenholtz and Van Hove con-
Messungen des Anlagerungskoeffizienten von Elektronen in Sauerstoff

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The attachment coefficient $\eta$ for the formation of negative ions in low energy electron swarms was measured over an $E/p$-range of 0.1 ... 30 V/Torr cm by a new and accurate method. Thus earlier measurements up to 54 Torr of other authors could be extended to 880 Torr.

The shape of the minimum in the $n/p$-curves between the three-body process ($e+2 \text{O}_2 \rightarrow \text{O}_2^-+\text{O}_2$) and the dissociative process ($e+\text{O}_2 \rightarrow \text{O}^-+\text{O}$) and its shift to higher $E/p$ with increasing pressure was measured. Behind the minimum a maximum at $E/p=14$ was found. Between the minimum and this maximum the dissociative process is predominant but the three-body process is still of influence. For $E/p > 14$ the $n/p$-values decrease with increasing $E/p$.

For the higher pressures above 44 Torr deviations from the relation $\eta \propto p^2$ were found for the three-body process. These deviations are discussed.

Es wird die Bildung stabiler negativer Sauerstoffionen bei der Drift von Elektronen im homogenen elektrischen Feld untersucht. Durch die Verwendung einer neuen, genaueren Meßmethode werden die früheren Ergebnisse anderer Autoren erweitert.

Der Anlagerungskoeffizient $\eta$ ist definiert als die Anzahl von Anlagerungen, die ein Elektron eines Elektronenschwarms im Mittel pro cm Driftweg erleidet. $\eta \, dz$ ist die Wahrscheinlichkeit der Anlagerung auf der Strecke $dz$.

Nach Chanin, Phelps und Biondi treten beim Stoß von Elektronen mit Sauerstoffmolekülen zwei unterschiedliche Anlagerungsprozesse auf, in denen die beiden negativen Ionen $\text{O}^-$ oder $\text{O}_2^-$ gebildet werden.

Sonderdruckanforderungen erbeten an Dr. R. GRÜNBERG, Institut für Angewandte Physik der Universität Hamburg, D-2000 Hamburg 36, Jungiusstr. 9.

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