Diagnostic Method for Measuring Ionization Rates in Shock Heated Gases

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The practicability of a method using two wires as transmission line for microwaves as a diagnostic method on a shock tube was investigated. The energy distribution around the wires was calculated which showed a high spatial resolution independent of the frequency for this method. The wires cause bow shocks because the flow is hypersonic which should be taken into account. The geometrical dimensions happen to be the same as those in a probe method used by Hobson, hence the disadvantages are comparable.

The ionization rate in shock heated gases, i.e. the variation of electron densities with time, can be measured by microwave methods. The microwave technique in the 4 mm region developed in our laboratory by Makios for similar measurements on a T-tube seems particularly suitable for this purpose. The microwave power is coupled to a Lecher system passing through the plasma. This method has the advantage of high spatial resolution.

In the region where atom-atom collisions predominate, the degree of ionization is less than $10^{-5}$ and the electron densities are less than $10^{12}\ \text{cm}^{-3}$, cf. 1. The suitability of this method at low frequencies is therefore investigated. Its advantages and disadvantages and the results of preliminary measurements are discussed. The Lecher wires have a diameter of 0.3 mm and a spacing of 4 mm. These dimensions happen to be the same as those in a probe method used by Hobson and co-workers for measurements of the ionization rate in the region of the atom-atom collisions, cf. for example 3. The advantages and disadvantages of their method and that discussed here show similarities. There is a good case for applying the two methods simultaneously.

Lecher System

As already shown theoretically by Mie in 1900 4, the wave guided by the wires in a homogeneous medium is transverse to the wire, and the field distribution governing the spatial resolution does not depend on the frequency. Fig. 1 shows the geometry of the Lecher system.

Using cartesian coordinates we have calculated the Poynting vector $\mathbf{S}$ which has a component only in the direction of the wave propagation and is proportional to:

$$S_z \sim \frac{4(b^2-a^2)}{(y^2+x^2)^{1/2} + 2[y^2+(x^2)](b^2-a^2) + (b^2-a^2)^2}$$

Fig. 2 shows the results.

The curves indicate concentration of energy near the wires. To prove this we have integrated the Poynting vector. Using bipolar coordinates $u, v$ (as in the calculation of Mie)

(cf. Fig. 1) : $u = \ln(r_0/r_1)$; $v = \varphi_1 - \varphi_2$.

We get for the power flow $N_W$

$$N_W = \int \int_{u, v} \mathbf{S} \cdot \mathbf{d}f = \int \int_{u, v} (\text{grad} u)^2 \mathbf{d}f = \int \int_{u, v} \mathbf{d}u \mathbf{d}v$$

and in the right half-plane (cf. Fig. 1):

$$N_W \approx \int \int_{u, v} \mathbf{d}u \mathbf{d}v = 2\pi u_0$$

with

$$u_0 = \ln\left[\frac{b+\sqrt{b^2-a^2}}{a}\right]$$

($u_0$ describes the circle of the wire surface).

The power in a circular ring $\pi(R^2-a^2)$ around a wire was calculated numerically in polar coordinates $R, \beta$ with the origin at the centre of a wire (cf. Fig. 1). (The limits of integration are then constant values.)

With the total power in the right half plane $N_W$ we get for the percentage of the total power flowing through the circular ring [Eq. (6) and (7)]:

$$100 \cdot \frac{N_W \text{ Ring}}{N_W} = 100 \cdot \frac{\int_0^{2\pi} R \cdot dR}{2\pi u_0} \int_0^{\beta_{\text{const}}} \frac{d\beta}{a} R \cdot dR$$

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4 G. Mie, Ann. Physik 2, 201 [1900].
Fig. 1. Geometry of the Lecher system.

Fig. 2. Values of Poynting vector around a wire.
with
\[ f(R, \beta) = \left[ R^2 + 2bR \cos \beta + b^2 \right]^2 + 2(b^2-a^2) R^2 \left( \sin^2 \beta - \cos^2 \beta \right) \]
\[ - 4(b^2-a^2) b R \cos \beta - 2(b^2-a^2) b^2 + (b^2-a^2)^2. \]

Fig. 3 shows the calculated values, i.e. the power distribution around the wire: Almost 70% of the input power is contained in a circular ring with, for instance \( R = 1.3 \) mm. The spatial resolution is then of the order of \( 1-2 \) mm (for every frequency).

Fig. 3. Power distribution around the wire from integrated Poynting vector. \( a= \) radius (not diameter!) of wire.

**Lecher Line and Plasma**

The frequency independence here allows application at low frequencies, i.e. small electron densities can be measured, this being of interest. If the plasma is homogeneous, the transversality of the wave enables a simple model to be used for calculating the interaction with the plasma: plane wave incident on plane plasma slab. Fig. 4 shows the measurable ranges calculated. The measurable electron densities are comparable to those in the probe method mentioned. The electron temperature can be calculated from the measured collision frequency.

**Transforming Components**

To apply this method the wave, e.g. from a rectangular waveguide, has to be coupled to the two-wire system. The coupling system used by MAKIOS\(^2\) radiates the greatest part of the input power. Coupling components with smaller radiative losses (40%) and with broad-band impedance matching were therefore developed.\(^5\) Figs. 5 a and 5 b show two types, type 5 b being particularly suitable for low frequencies. (The Lecher line does not radiate any power, as was checked by measurements.) Because of the geometrical shape of the transformers which is necessary for wave transformation the condition for impedance matching is automatically satisfied at the same time.

![Fig. 5. Transformers to couple the power onto the lecher system; a) from 3 cm waveguide; b) from coaxial system.](image)

**Preliminary Measurements**

To test the method, measurements were made on a diaphragm shock tube (inside diameter 10 cm). The driving gas was hydrogen (20 atm), the working gas argon (e.g. 0.5; 1; 2 torr); the shock Mach number at the measuring site (4.4 m from the diaphragm): \( M_s = 10.1; 9.3; 8.7 \) (measured with thin-film gauges). The impurity level was still too high.

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The measuring chamber was made of epoxy resin with thin-film gauges and perspex windows (thickness: half the wavelength in perspex) admitting the wires. For the microwave measurements we used an interferometer working at 8.58 Gc/s with an output signal proportional to

$$E_1 E_2 \cos \varphi$$

where $E_1$ and $E_2$ are the amplitudes of the waves passing through the measuring and reference branches respectively, $\varphi$ being the phase difference of the waves.

Fig. 6 shows a typical oscillogram, Fig. 7 an example of the variation of the electron density with time (only the phase reversal points are evaluated). The lower values are already in the region where atom-atom collisions predominate.

The measured collision frequency was in this case $2 \cdot 10^{10}$ c/s. Considering only electron argon collisions one can calculate the electron temperature using for instance, the collision cross sections given by Devoro $^6$. The result is a value which is about twice the gas temperature behind the shock front in the case of unperturbed gas flow. For physical reasons this seems impossible. On the other hand, Makios $^2$ also measured such excessively high values of collision frequencies behind the shock wave in a T-tube.

**Bow Shocks**

The wires cause bow shocks because the flow is hypersonic. Fig. 8 b shows Mach-Zehnder interferograms of the bow shock around one of the wires, Fig. 8 a the geometric configuration, Fig. 8 c the evaluated density variations (from an unpublished investigation of KLINGENBERG and ZIMMERMANN). The bow shock is 1.3 mm from the wire. Fig. 3 shows that the measurements are confined practically to the region behind the bow shock. The density change averaged over the circular ring with $R = 1.3$ mm is about 1.5, the gas temperature change averaged is perhaps a factor of 2. On passing through the ring (in about 1 $\mu$sec) a particle undergoes about $10^4$ collisions at a collision frequency of $10^{10}$ c/s. These changes have to be taken into account and may explain at least partly the high values of the collision frequency measured. Owing to the shock wave and to the boundary layer around the wire the plasma around the wire is no longer homogeneous.

**Discussion**

The Lecher wire method is practicable in principle and particularly suitable in the lower frequency range. The high energy concentration around the wires independent of frequency is an advantage from the point of view of spatial resolution but enables measurement only in the region between the bow shock and the wire. The bow shocks are unavoidable since due to technical difficulties the dia-

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meter of the wire cannot be taken smaller than the mean free path of the gas particles. This has to be taken into account. The strong inhomogeneity of the electromagnetic fields in conjunction with the inhomogeneity of the plasma (boundary layer) raises the question whether the wave remains plane i.e. it is questionable whether the simple model of plane wave and plane plasma slab can at all be used.

Despite of these disadvantages it seems useful to compare this method with Honson's probe method, and the investigations should be continued along these lines.

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