Investigation of a Caesium Plasma Diode Using an Electron Beam Probing Technique

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The plasma in a plane caesium diode with a hot emitter and a cold collector was investigated experimentally with a ribbon-shaped electron beam. The ribbon beam is projected through the diode at an angle of 45 degrees to its axis and allowed to strike a fluorescent screen. Variations in the axial electric field of the diode cause the ribbon beam to be distorted. The image of the distorted beam as seen on the fluorescent screen then constitutes a plot of the axial electric field along the axis of the diode.

The field plots so obtained are compared with a theory in which the collisions of the charge carriers are neglected. By means of this comparison it is possible to evaluate the neutralization parameter, the plasma density, and an average drift energy of the charge carriers.

The results show that the theory correctly describes the different modes of the potential distribution and especially the transitions between modes of operation as long as the diode is free of oscillations.

The stability of the different possible static potential distributions was also investigated. It was found experimentally that the system is unstable if the electron emission is less than the ion emission and the collector potential is positive.

Since the ionization of atoms of low ionization energy on hot surfaces with high work function was discovered by LANGMUIR and KINGDON in 1925, widespread use has been made of contact ionization.

Plasmas produced by contact ionization can be maintained without currents. This is a great advantage because currents cause many kinds of instabilities. Such plasmas are therefore useful for investigating the conditions necessary for thermonuclear fusion. The fact that a deuterium plasma with a temperature of more than 100 000 K can be simulated by a caesium plasma (on account of the large


2 This work was performed under the terms of the agreement on association between the Institut für Plasmaphysik and EURATOM.

gyroradii) with a temperature of only 2000 °K is also of advantage for the investigation of a plasma in a magnetic field.

In thermionic energy converters there is a high concentration of electrons in front of the cathode. The space charge of these can be compensated by producing positive ions by contact ionization.

In such experiments it is important to know the electric potential distribution in the sheaths in front of the cathode and in the plasma.

Our object was to investigate the potential distribution in a diode extremely similar to a thermionic energy converter. Electrons and ions are only produced at one electrode, the heated emitter. The other electrode is kept cold so that it collects all the electrons and ions reaching it.

The diode was constructed and operated in such a way as to keep as simple as possible the theory required to describe it: plane electrodes with a separation small relative to their diameter reduce this to a one-dimensional problem. The gas density chosen was so low that electrons and ions in the volume undergo hardly any collisions. In the theory the potential distribution is then found by determining the space charge density of the electrons and ions as a function of position for the one-dimensional, collisionless case and integrating the Poisson equation.

Simple expressions would be obtained for the space charge density if the charged particles could be treated as monoenergetic. This simplification means, however, that the number density of one type of carrier increases and even becomes infinite at the inversion point if the carriers buck against a potential barrier. Allowance has therefore to be made for the velocity distribution of the emitted carriers, it being assumed that this is a Maxwell distribution, whose temperature agrees with that of the emitter. It is then possible to give analytical expressions for the carrier densities as a function of the potential, but the potential distribution has to be calculated numerically.

A monotonic potential distribution is given by the theory only in part of the complete range of parameter values. The space charges are actually concentrated in narrow regions near the emitter and collector, and so we are justified in talking of sheaths. In other ranges of the parameter values we have in front of the emitter a double sheath with an extreme value of the potential, while at the collector there is a single sheath. In addition, however, there is also range of parameters in which the plasma potential is not constant, but oscillates spatially.

In Chap. 1 first the parameter ranges for the various modes of the potential distribution are separated from one another, and then individual examples of the potential distribution are calculated.

The electric fields were measured with the aid of an electron beam that causes hardly any perturbations at low beam current densities and high acceleration voltages. A new technique in which the beam passes through the diode in the form of a ribbon is used. This enables the static and dynamic field distributions to be projected on to a fluorescent screen.

The experiments proved that the potential distribution modes postulated by the theory actually exist. The transitions between the modes also agree with the theory.

In some parameter ranges, oscillations of both the diode current and the field patterns occurred. The frequency of these oscillations is in the ionic sound region. They have already been observed in many other laboratories. Various attempts at explaining the origin of these are described in the literature. They are said to be due, for instance, to ion oscillations in a potential well formed in front of the emitter by electrons or to a two-stream instability. But these and other approaches are at variance with the static theory of experiments. A more recent theory has been more successful. It is based on the fact that the ions react much more slowly to a perturbation than the electrons because of their relatively large mass. The electron density distribution, as described by the static theory, may therefore be unstable. The authors of this theory were unable, however, to define theoretically the parameter range in which the instability occurs.

The pronounced non-linearity of the diode oscillations postulated by the authors is shown in our experiment by distinct structures in the field patterns.

1. Static Potential Distribution in the Diode

1.1. Theory

The diode in question comprises a hot, plane electron-and-ion emitter (cathode) and a cold, plane collector (anode). A model is shown in Fig. 1. The
two electrodes are placed parallel, opposite one another. It is possible to maintain an arbitrary potential difference between them.

![Diagram of caesium diode](image)

**Fig. 1.** Principle of the caesium diode.

The theory is simplified by making the following assumptions:

1) Emitter and collector are plates of infinite extent. The potential at each of the two surfaces is constant.
2) Electrons and ions are emitted with a Maxwellian velocity distribution. Their temperature is equal to that of the emitter.
3) Charged particles do not undergo any collisions in the volume between emitter and collector.
4) Charged particles are not reflected from the surface of either the emitter or the collector.

Taking the ions as an example, we shall now show the dependence of their velocity distribution on the potential and on the position relative to extreme values of the potential.

As postulated, the velocity distribution of the evaporating ions right on the emitter surface is

\[
f(v) = n_{i0} \sqrt{\frac{2 m_i}{\pi k T}} \exp\left(\frac{-m_i v^2}{2 k T}\right), \quad v > 0
\]

where \( m_i \) is the mass of the ions, \( v \) their velocity, \( k \) the Boltzmann constant and \( T \) the emitter temperature. Only the ions moving away from the emitter are taken into account in the number density \( n_{i0} \).

A contribution to the total ion density near the emitter surface may also be made by the ions reflected by a potential increasing in the volume.

We now look for static potential distributions, i.e. we have to solve the time-independent Vlasov equation, which has to be one-dimensional and collisionless as postulated:

\[
v \cdot \frac{\partial f}{\partial x} - \frac{e}{m_i} \cdot \frac{\partial f}{\partial v} = \frac{\partial U}{\partial x} \cdot \frac{\partial f}{\partial v} = 0.
\]

Here \( x \) is the distance from the emitter, \( U \) the electrical potential relative to that of the emitter surface.

The Vlasov equation is solved by

\[
f(x, v) = \begin{cases} 
  n_{i0} \sqrt{\frac{2 m_i}{\pi k T}} \exp\left(\frac{-m_i v^2}{2 k T}\right) \cdot \exp\left(-\frac{e U(x)}{k T}\right) \\
  0
\end{cases}
\]

i.e. the velocity distribution in the volume deviates from a Maxwellian distribution by a changed density factor depending on the potential \( U \) (Boltzmann factor) and by regions in velocity space, where the distribution function is zero. The extent of these regions must be determined for each individual type of potential distribution.

Fig. 2 gives examples of the ion velocity distribution at individual points of characteristic potential forms.

![Potential distribution and corresponding ion velocity distribution](image)

**Fig. 2.** Potential distribution and corresponding ion velocity distribution.

Since the carrier density is now known, the one-dimensional Poisson equation

\[
\frac{d^2 \eta}{d \xi^2} = \frac{n_e}{n_{i0}} - \alpha \cdot \frac{n_i}{n_{i0}}
\]

can be integrated. This formula uses the normalization

\[
\eta = eU/kT \quad \text{for the potential},
\]

and

\[
\xi = \frac{x}{\sqrt{\epsilon_0 k T / (e^2 n_{i0})}} \quad \text{for the distance from emitter}.
\]
The unit of length \( \sqrt{\epsilon_0 k T / (e^2 n_e_0)} \) represents a kind of Debye length which, however, is not related to the plasma density, as is usually the case, but to the number density \( n_e_0 \) of the evaporating electrons near the emitter surface. The new parameter introduced \( \alpha = n_{i0} / n_e_0 \) states how much the emitter beams deviate from charge neutrality.

The Poisson equation can be integrated in closed form once
\[
\frac{1}{2} \left( \frac{d\eta}{dz} \right)^2 = \int \left( \frac{n_c}{n_{e0}} - \alpha \frac{n_i}{n_{i0}} \right) d\eta = F(\eta).
\]
Like \( n_i \) and \( n_e \), the analytical forms of \( F(\eta) \) depend on both the shape of the potential distribution and on the position relative to extreme values or zeros of the potential.

When the external parameters:
- the normalized collector potential \( \eta_c = eU_c / (kT) \) relative to the emitter potential,
- the neutralization parameter \( \alpha = n_{i0} / n_e_0 \),
- the normalized distance \( \xi_c = x_c / \sqrt{\epsilon_0 k T / (e^2 n_e_0)} \) between emitter and collector
are given, it is unfortunately not clear at the outset what monotonic or nonmonotonic form the potential distribution will take in principle. First a shape has to be assumed and then its bounding surface in parameter space has to be defined afterwards.

There are essentially three possible types of potential shapes: monotonic, nonmonotonic with a double sheath at the emitter and nonmonotonic with a wavy plasma potential. Each type must be considered individually, and its bounding surface in parameter space has to be calculated numerically.

Either one of the parameters:
- the normalized field intensity \( \eta'(0) \) at the emitter,
- the normalized potential \( \eta_{\text{max}} \) of a possible maximum,
- the normalized potential \( \eta_{\text{min}} \) of a possible minimum,
should be chosen such that the integral \( \int d\eta / \sqrt{2 F(\eta)} \) over the entire potential range gives exactly the normalized separation of the emitter from the collector. The numerical computation shows that the potential curves in the sheath regions depend very little on this separation, provided that the separation is large compared with the modified Debye length.

The range of occurrence of the different types of potential distribution in the two-dimensional parameter space \((\alpha, \eta_c)\) remains to be determined. A plot of the bounding lines as obtained by numerical calculation is given in Fig. 3. In this semilog plot (log \( \alpha - \eta_c \)) all curves are symmetric with respect to the point \( \alpha = 1, \eta_c = 0 \).

The regions of monotonically increasing and decreasing potential open to the right (marked 1) and left \((1')\) respectively. If we continue to move in a clockwise direction, we arrive at modes with a minimum or maximum in front of the emitter. These are adjoined by the region of those unusual modes whose plasma potential is not constant, but oscillates spatially.

The monotonic modes have two sub-ranges. In the larger one \((1 A \text{ or } 1'A)\) there is a single sheath at the emitter, in the smaller one \((1 B \text{ or } 1'B)\) a double sheath. The oscillatory modes also have two sub-ranges. In the one the potential always remains positive \((3 A)\) or negative \((3'A)\) relative to the emitter potential. In the other sub-range \((3 B, 3'B)\) the potential first passes through zero before oscillation sets in.

In our experiment the potential difference \( \eta_c \) between collector and emitter can be varied much more easily than the neutralization parameter \( \alpha \). The potential distribution for \( \alpha = 0.3 \) and various collector potentials were calculated. They are plotted in Fig. 4. The corresponding electric field curves are plotted in Fig. 5.

Just after the original manuscript had gone to press it was discovered that a similar diagram is given by R. G. McIntyre, Proc. IEEE 51, 760 [1963].
1.2. Description of the Apparatus

The apparatus used for all the experiments described here is sketched in Fig. 6.

The main section of the apparatus consists of an iron block containing a water-cooled, nickel-plated iron pipe. The inner diameter of the pipe is 80 mm. It is continuously pumped out by two 120 litres/sec oil diffusion pumps. A tantalum emitter is inserted from one side of the pipe, a cooled stainless steel collector from the other, both these being 32 mm in diameter. Both the emitter and the collector can be moved in the axial direction. They are electrically insulated from the rest of the apparatus by teflon flanges.

Outside the pipe is a caesium oven, which can be heated to 250 °C with oil from a thermostat. Caesium vapour from the oven is radiated on to the emitter from an opening in the pipe. The construction of the emitter and the caesium oven is based mainly on a design by Guilino. The difference in temperature between the centre and edge of the emitter surface was 50 °K when the temperature in the centre was 2500 °K.

The emitter is heated by electron bombardment from a ring cathode. The heating beam has typical values of 1500 V for the acceleration voltage and 1 A for the beam current. The heating power was obtained from a power stabilizer designed by Maischberger and Stein-
HAUSEN. Stabilizing the power to ±0.5% ensured an emitter temperature constant within ±0.1°C as long as the grey factor of the emitter surface did not vary. The surface of a new emitter was dull grey. After many hours of heating, monocrystals 1 to 2 mm in diameter formed on the surface. The surface then became shiny. Further heating does not seem to cause any appreciable change in the grey factor, and so it was easy to keep the temperature constant. The emitter temperature was measured with an optical pyrometer. A grey factor of 0.4 was used to correct the values displayed by the pyrometer. The temperature thus obtained may involve a systematic error of ±50 °K, relative temperatures are accurate to within ±0.5 °K.

Before the measurements were made, the emitter and caesium oven were preheated for about an hour. The caesium capsule was then broken open and another hour allowed to pass. In this way, it was ensured that all cold surfaces inside the pipe were covered with caesium and that stationary conditions prevailed during the measurements. Caesium was then supplied by the emission from the surface (∼10^-6 to 10^-4 Torr caesium vapour pressure) and by the additional vapour jet from the oven. Compared with the method using a closed volume and heated walls, this type of operation has the advantage that caesium losses through the electron beam diaphragm are continuously replaced. It is rather difficult, however, to obtain exactly reproducible conditions.

The electron beam was produced in a cathode system from a Siemens electron microscope. The high voltage (5 to 60 kV) was derived from a Zeiss stabilized high-voltage unit supplying currents of up to 0.5 mA. The electron beam first passed through the centre of an electric quadrupole, then through a slit diaphragm 4 × 0.03 mm, entering the diode as a narrow ribbon-shaped beam. This ribbon beam finally impinged on a fluorescent screen above which a thin wire was stretched to provide a reference line. The image appearing on the screen was photographed with a 40 mm lens and then enlarged.

Dirt present on the slit diaphragm became charged and caused serious distortion of the beam image. This proved troublesome at first, and then the diaphragm holder was designed to allow the diaphragm to be baked during operation by passing current through. Broadening of the electron beam due to power line hum originating primarily from the ring cathode inside the emitter was remedied by heating the ring cathode with half-wave rectified current and switching on the beam only during the rest period. The pulse duration could be regulated by means of a Tektronix T plug-in unit from 8 msec down to the shortest time required.

1.3. Measurement of the Field Intensity with the Ribbon Electron Beam

The method of measuring electric fields in plasmas by passing an electron beam through and determining the amount of deflection is some fifty years old (Thomson, Aston). Since then it is being used more and more. Some authors were content with point-by-point measurement of the deflection on a screen, others elaborated the method to such an extent that it became possible to use a recorder. The amplitude of alternating fields or their time dependence was also determined. What these methods have in common is that the system producing the field intensity to be measured is taken past the spatially fixed, threadlike electron beam, the field being measured at individual points at various times.

The present approach represents an innovation because a spatial field distribution is projected straight on to the fluorescent screen (Fig. 7). A snapshot of this trace gives both the distribution of the steady field and, by means of its envelope, of the amplitude of a possible alternating field.

For this purpose the beam was first passed through an electric quadrupole. This has the same effect as two crossed cylindrical lenses, one focusing in the direction of the line connecting the negative poles...
and one defocusing in the direction of the positive poles. A circular beam cross section is thus distorted by the quadrupole into an ellipse which keeps getting narrower and narrower all the way down the beam path. The voltage at the poles of the quadrupole is set so that the beam cross section on the screen degenerates to a line. The quadrupole itself is adjusted so that this line runs parallel to the emitter surface.

A slit diaphragm is then inserted between quadrupole and diode, its longitudinal axis forming an angle of 45° with the emitter surface. This produces a ribbon beam which in the diode is still at an angle of approximately 45° to the emitter surface. By the time it reaches the screen, however, it is parallel to the emitter, thus fixing the base line. The distance $y$ (see Fig. 7) taken by an electron to reach the screen is thus proportional to the distance from the emitter at which the electron in question passes through the diode.

For reasons of symmetry the field intensity inside the diode is parallel to the diode axis ($x$-direction). It superposes on the motion of an electron a velocity component in the $x$-direction. The deflection caused by the field intensity is thus perpendicular to the base line on the screen. That is, we obtain on the screen a plot of the field distribution in rectangular coordinates.

The beam width on the screen was equivalent to a field intensity of 6 V/cm. The sensitivity of the field intensity measurement was about 1 V/cm.

A parallel method is described in a paper by Dow\textsuperscript{28} in which radial fields in a PIG discharge were measured with a ribbon beam. In this case the direction of the electric field did lie in the plane of the ribbon beam, but the $E \times B$ drift caused a deflection in the perpendicular direction and hence produced an image of the field distribution on a screen. The author's opinion was, however, that it was not possible to make a quantitative evaluation.

1.4. Experimental Results and Comparison with the Theory

A set of field curves plotted for a constant emitter temperature (2400 °K) and a constant caesium supply, but for various collector voltages, is shown in Fig. 8. The current-voltage characteristic of the diode gives the neutralization parameter $\alpha = 0.7 \pm 0.1$ and the plasma density $n = 7 \times 10^9$ cm$^{-3}$. For a markedly negative collector voltage $U_c$, we obtain a pure electron sheath at the emitter and an extended ion sheath at the collector. If $U_c$ is increased, the electron

\[ U_c = -0.84 \text{ V} \]

\[ U_c = -1.45 \text{ V} \]

\[ U_c = -1.75 \text{ V} \]

\[ U_c = -1.95 \text{ V} \]

\[ U_c = -2.3 \text{ V} \]

\[ U_c = -2.5 \text{ V} \]

\[ U_c = -2.7 \text{ V} \]

\[ U_c = -3.0 \text{ V} \]

\[ U_c = -3.5 \text{ V} \]

\[ U_c = -4.3 \text{ V} \]

\[ U_c = -5.0 \text{ V} \]

Fig. 8. Experimental field curves for $\alpha = 0.7$, $n = 7 \times 10^9$ cm$^{-3}$, $T_{\text{EM}} = 2340$ °K.

\textsuperscript{28} D. G. Dow, J. Appl. Phys. 34, 2395 [1963].
sheath at the emitter at first remains unchanged, as predicted by the theory, and only the ion sheath at the collector becomes thinner. Finally, the collector sheath disappears and, again in agreement with the theory, a spatially oscillating field is produced. Near the emitter the voltage amplitude is \((0.1 \pm 0.02) \, kT/e\) (in theory up to \(0.4 \, kT/e\)). As the distance from the emitter increases, the waviness decreases, although the theory predicts that the field should be purely periodic. If the collector voltage is increased further, we observe the theoretically predicted transition to a monotonic potential distribution in which there is a double electron-ion sheath at the emitter and an electron sheath at the collector.

As the collector voltage is raised even higher, the electron sheath at the collector should become thicker according to the theory. Diode oscillations occur instead. These are dealt with in greater detail in Chapter 2. As long as no diode oscillations are present, there is thus a transition of the modes from \(1'\Lambda\) via \(3'\) to \(1\Lambda\) in agreement with the theory (cf. Fig. 3).

Another set of field curves is shown in Fig. 9. These were plotted for the case of an excess of ions \((\alpha = 9 \pm 1)\). The emitter temperature was \(2100 \, ^{\circ}\)K, the plasma density \(8 \times 10^9 \, \text{cm}^{-3}\). When the collector potential is strongly negative, one obtains at the collector a wide ion sheath and at the emitter a double ion-electron sheath with a potential maximum. Such a non-monotonic potential distribution in a contact ionization plasma was observed for the first time in our apparatus \(^{29}\). The drift energy of the ions in the plasma was found from the field curve to be \((2.8 \pm 0.5) \, kT\) (in theory \(2.1 \pm 0.15\) for \(\alpha = 9 \pm 1\)).

Increasing the collector voltage again results, in particular, in a reduction of the ion sheath at the collector and finally causes it to disappear altogether. Here, too, a slightly wavy field with a voltage amplitude of \((0.2 \pm 0.05) \, kT/e\) (in theory up to \(2 \, kT/e\)) then results, with the double sheath at the emitter persisting. As the collector voltage is increased even more, the double sheath becomes a single ion sheath, while a weak electron sheath forms at the collector. According to the static theory a further increase in the collector voltage should cause this electron to grow. But oscillations occur instead.

The static potential distributions thus show here the mode transitions \(2' \rightarrow 3 \rightarrow 1\Lambda\) predicted by the theory.

Fig. 10 * shows original field curves recorded for a neutralization parameter \(\alpha = 30 \pm 10\), a plasma density \(n = 8 \times 10^9 \, \text{cm}^{-3}\) and an emitter temperature of \(2000 \, ^{\circ}\)K. The mode transitions are the same as those in Fig. 3.

Finally, four original pictures of wavy field curves taken for \(2200 \, ^{\circ}\)K, \(\alpha = 0.5\) and \(n = 4 \times 10^9 \, \text{cm}^{-3}\) are shown in Fig. 11.

The measured field curves thus prove the existence of the monotonic and non-monotonic potential distribution modes postulated by the theory. The mode transitions are also correctly described by the theory.

### 1.5. Comparison with the Literature

The theory of the potential distribution in a diode goes back to Langmuir \(^{30}\). For the case of pure electron emission he took the velocity distribution of the emitted electrons into account and arrived at the surprising result that a potential minimum may form in front of the cathode.

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* Figs. 10, 11, 13, 14 on p. 1064 a, b.

Fig. 10. Experimental field curves for $\alpha = 30$, $n = 8 \times 10^8 \text{cm}^{-3}$, $T_{EM} = 2010 \degree \text{K}$.

Fig. 11. Experimental field curves for $\alpha = 0.5$, $n = 4 \times 10^8 \text{cm}^{-3}$, $T_{EM} = 2270 \degree \text{K}$. 

Zeitschrift für Naturforschung 22 a, Seite 1064 a.
Fig. 13. Oscillations of the field intensity (left) and diode current for $\alpha=5$, $n=2\times10^{10} \text{ cm}^{-3}$, $T_{EM}=2200 \degree \text{K}$.

Fig. 14. Field oscillations for $\alpha=10$, $n=5\times10^9 \text{ cm}^{-3}$, $T_{EM}=1970 \degree \text{K}$. 
This basic conception of including the velocity distribution was extended by Auër \(^{31}\) to the collisionless diode with electron and ion emission of the cathode. He has also already given the three, in principle, different potential distributions (see Fig. 3) which can occur in the diode. McIntyre \(^{32}\) extended the theory by a whole series of quantitative solutions, and the problem is also dealt with in the papers \(^{33,34}\).

Characteristic of this collisionless theory is the occurrence of wavy plasma potentials. The existence of these has often been called in question in the literature. Franklin \(^{35}\), for instance, denied them any physical reality, while Maëv \(^{36}\) concluded that the diode is in an unsteady state. A few Russian authors \(^{37-39}\) were of the opinion that the potential wells fill up with slow carriers captured by the potential wells as a result of rare collisions. They tried to solve the Boltzmann equation for this case, but finally they adopted for the distribution function of the captured carriers a Maxwell–Boltzmann form which states that the gaps in the distribution function of the primary charge carriers are largely filled. As expected, there are no wavy plasma potentials in this case.

Our experiments now clearly show such wavy plasma potentials. They are therefore direct proof of the validity of the collisionless theory.

1.6. Application of the Theory to Diode Operation without Current

We wanted to apply the results to Q machines in single-emitter operation. Caesium plasmas are produced in Q machines by shooting caesium vapour at a hot emitter, as in our case. Since high magnetic fields in the axial direction and long plasma columns are used in Q machines, the one-dimensional treatment for an infinite emitter-collector separation is justified. Because the properties of the plasma are strongly affected by currents in ways not readily understood, it has become customary to keep the emitter free of current in Q machine operation. In order to describe the given plasma theoretically, the floating potential \(\eta_{ct}\) of the collector at which the ion and electron currents just compensate one another has to be found for a fixed neutralization parameter \(\alpha = n_{o0}/n_{a0}\). From this the plasma potential \(\eta_p\) is then calculated and the corresponding drift energy of the ions and electrons determined.

In the monotonic region \(\alpha'\) of the \(\alpha - \eta_c\) plane (Fig. 3), the condition that the current densities of the ions and electrons be equal gives us

\[
\eta_{ct} = \ln \alpha - \frac{1}{2} \ln \left(\frac{m_i}{m_e}\right)
\]

with \((\frac{1}{2}) \ln (m_i/m_e) = 6.2\) for caesium. The plasma potential \(\eta_p\) is obtained from the zero of the Poisson equation because of the quasi-neutrality condition. The drift energy of the ions, in \(kT\) units, is then \(\varepsilon_d = -\eta_p\). The electrons have no drift velocity superposed on the thermal motion.

In the region \(\alpha'\) with a single potential maximum \(\eta_{max}\) we obtain

\[
\eta_{ct} = \ln \alpha - \frac{1}{2} \ln \left(\frac{m_i}{m_e}\right) - \eta_{max}
\]

Here we have thus to find that combination of \(\eta_{ct}\), \(\alpha\), and \(\eta_{max}\) which satisfies the appropriate Poisson equation and boundary conditions. The drift energy of the ions is \(\varepsilon_d = \eta_{max} - \eta_p\). There is no electron drift in this case either.

The oscillatory region \(\alpha''\) need not be considered. It is reached only for absolutely uninteresting values of the boundary conditions \((\eta_{ct} \approx -2 \times 10^2, \alpha \approx 5 \times 10^{-3})\) if the collector is not drawing current.

The floating potential \(\eta_{ct}\) for caesium as a function of \(\alpha\), the relative plasma potential \(\eta_p\) and the drift energy of the ions \(\varepsilon_d = -\eta_p\) or \(\varepsilon_d = \eta_{max} - \eta_p\) are given in Fig. 12. The result is surprising because it means that in diode operation without current the electrons never have a drift, whereas the ions always do no matter how great the excess of ions at the emitter may be.

The connection between plasma density \(n_p\) and ion saturation current density \(e_j\), which are measured with, say, a Langmuir probe, is then

\[
n_p/e_j = \sqrt{2\pi m_i/kT} \cdot \frac{1}{2} \exp(\varepsilon_d) \cdot \left[1 - E(\varepsilon_d)\right]
\]

with

\[
E(\varepsilon) = \frac{2}{\sqrt{\pi}} \int_0^{\sqrt{\varepsilon}} e^{-t^2} dt
\]

\(^{31}\) P. L. Auër, J. Appl. Phys. 31, 2096 [1960].
\(^{34}\) K. G. Hernqvist and F. M. Johnson, Advanced Energy Conversion 2, 601 [1962].
\(^{35}\) R. N. Franklin, J. Electron. Control 9, 385 [1960].
\(^{36}\) S. A. Maëv, Soviet Phys.—Techn. Phys. 8, 842 [1964].
\(^{39}\) F. G. Baksht, Soviet Phys.—Techn. Phys. 9, 713 [1964].
as opposed to the case of a Maxwellian velocity distribution complete on both sides for which
\[ n_{ij} |_{\text{Maxwell}} = \sqrt{2 \pi} \frac{m_j}{kT} \]
is valid. The correction factor
\[ K = \frac{2}{\exp(\varepsilon_d) [1 - E(\varepsilon_d)]} \]
is included in Fig. 12. This shows that under floating conditions the Maxwell density is too high by a factor of at least 3.

This statement is confirmed by the measurements of Levine et al.\(^{40}\). In a Q machine in single-emitter operation they found that the plasma density determined in the usual way from the ion saturation current is too high by a factor of 2–3 compared with that obtained by other methods.

### 2. Oscillations in the Diode

#### 2.1. Experimental results

Pronounced oscillations of the diode current have been dealt with in the literature so often\(^{2,5,41–46}\) that we need not describe the results of our measurements in detail. The characteristics of the current oscillations listed in the literature were confirmed:

1) The oscillations occur only when the neutralization parameter \( z = n_{ij0}/n_{e0} \) is greater than about 0.5.

2) With increasing collector voltage the oscillations do not set in until the electron saturation current has been attained. At the same time the current strength usually starts to drop again.

3) The cycle of the oscillations is of the order of the transit time of the ions through to diode.

4) The frequency of the oscillations increases with rising collector voltage.

5) The form of the oscillation may be distinctly non-sinusoidal.

As expected, we were able to verify the existence of these oscillations with the ribbon electron beam. The records of the field oscillations were much the same as those of the current oscillations. The current oscillations sometimes presented a diffuse picture on the oscilloscope, but they often had a definite form giving a clear oscillogram. Similarly, the record of the field intensity was either blurred or showed clear structures. The more the current oscillations deviated from a sinusoidal form, the more pronounced were these structures.

This analogy is illustrated in Fig. 13 by juxtaposing simultaneous records of the field pattern and oscillogram. The structures of the field patterns show the pronounced non-linearity of the field oscillations.

Another set of records is given in Fig. 14. The structures are again very prominent. The base line included in the photograph shows that the field intensity passes through zero in the opposite direction and that the potential becomes more negative than the emitter for a moment. This proves that the decrease of the mean direct current is caused by electrons being reflected to the emitter during part of the oscillation period.

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2.2. Discussion of Various Theories

The literature contains quite a number of theories that try to explain the origin of the diode oscillations. Using the experimental and theoretical results obtained in this work, we shall now appraise the most important of these theories:

1) Some theories state that the ions oscillate in a potential minimum formed by excess electrons, modulating the electron current as a result. One now finds that oscillations only occur if there is an excess of ions (or at least if \( z > 0.5 \)) and if the collector potential is positive. According to our theory, however, there is no potential minimum in front of the emitter (cf. Fig. 3) under these conditions. If, on the other hand, a potential well for the ions forms in the presence of strong electron emission, the diode is stable.

2) The occurrence of spatially periodic states is supposed to cause oscillations, but our experiments demonstrate that wavy potentials distributions are stable throughout. According to Fig. 3, moreover, the region of these wavy potentials is strictly limited in the direction of positive collector potentials. The oscillations would then simply have to cease above a certain positive collector voltage, but this is not the case.

3) Strong ion emission and the accompanying electron drift have suggested the existence of a two-stream instability in which ion waves are excited. This instability would then have to be present, however, in the case of strong electron emission as well because, according to our theory, there is also electron drift, provided the collector potential is positive. Our conditions are, however, stable.

4) Owing to the drop in the mean diode current intensity at the onset of the oscillations, these have been regarded as the result of a negative resistance of the diode. A negative resistance is not obtained, however, from the static theory.

5) Because of the large difference in mass between the ions and electrons, the spatial electron density distribution reacts much more readily to a perturbation than does the ion density distribution. According to Norris and Burger, the time-independent Vlasov equation together with the Poisson equation may yield solutions for which the electron density distribution is unstable if the ion density distribution is fixed.

Norris and Burger are of the opinion that the oscillations set off by the instability are characterized by the fact that variations of the electron density distribution, whereby the ion density distribution remains virtually unchanged, alternate with variations of the ion density distribution. Viewed as a whole, this oscillation becomes distinctly non-linear. The oscillation period must be of the order of the ion flight time.

For typically unstable cases, Burger used a computer simulation technique to calculate the potential distribution and diode current as a function of time, achieving close agreement with his experiments. His theory did not enable him, however, to state why and when the diode is unstable.

Norris, on the other hand, gave a stability criterion, but it is difficult to apply and extensive numerical calculations are necessary in every single case. The unwieldiness of his criterion is typified by the fact that Norris gave examples of stable and unstable potential distributions which are in direct contrast to the experiment. Further theoretical investigation should therefore be made to determine the cases in which the static solution is unstable.

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