Parametrization of Backward $\pi p$ Scattering

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(Z. Naturforschg. 21 a, 1193—1194 [1966]; received 21 March 1966)

Dedicated to Professor Dr. W. GENTNER on the occasion of his 60th birthday

Consider the process in Fig. 1. If a particle E exists, such that the quantum numbers are conserved at the vertices, the cross-section is enhanced for small momentum transfers. To be more quantitative: the closer the square of the four-momentum transfer $u = (p_\text{c} - p_\text{f})^2$ approaches the pole at $u = -m^2$, which lies in the non-physical region, the larger is the cross-section. Whether this description can be formulated in a satisfactory way so as to give detailed quantitative agreement with experiments has yet to be shown. More accurate experiments are certainly welcome.

I am going to discuss the situation when a meson A is incident on a proton B, and the meson D is scattered backwards while the recoil proton C goes forward. The exchanged particle E is then a baryon, which for $\pi^- p$ scattering can have isotopic spin 1/2 or 3/2. In the case of $K^- p$ scattering a hyperon can be exchanged, while for $\pi^- p$ there is no known particle E to be exchanged. There are unfortunately so far no conclusive experiments of Kp back scattering.

In an early experiment at the CERN Proton Synchrotron we looked at the backward elastic and inelastic scattering of $\pi^-$ on protons in the momentum region 1 to 2.5 GeV/c. The experiment was aimed at looking for meson resonances produced, as it were, at large momentum transfers. The forward recoiling proton was momentum analyzed, and detected with a differential Cerenkov counter. The backward elastic scattering was found to be 0.2 mb/ster at 1.5 GeV/c, in agreement with later data. The two-body production of $\pi^- p$ was found to be $\leq 10 \mu$b/ster.

In a recent experiment we used spark chambers to record the trajectories of the three particles involved in elastic scattering. The kinematic constraints are in this case much more severe, and we could therefore use a detection system with a large aperture. This experiment, and an experiment at the Brookhaven National Laboratory (BNL), give at the moment the most useful information on $\pi p$ backward scattering. There does not exist any information on polarization effects in backward scattering at high energies, so we will neglect spin dependence. The scattering amplitude is then a function of the Lorenz invariants $s$ and $u$ only, where

$$s = (p_\text{i}^2 + p_\text{f}^2)^2 = (p_\text{i}^2 + p_\text{f}^2)^2 = (E_\pi^* + E_\pi^* - E_{\pi^*} - E_{\pi^*})^2,$$

$$u = (p_\text{i}^2 - p_\text{f}^2)^2 = (p_\text{i}^2 - p_\text{f}^2)^2 = (E_{\pi^*} - E_{\pi^*} - 2 k^2 (1 - \cos \Theta^*))$$

$p$ are the four-momenta of the particles involved, $i$ indicating initial and $f$ final particles, $E^*$ and $k^*$ are energy and momentum in the centre of mass. $\Theta^*$ is the angle between the primary pion and the recoil proton. Due to the mass difference between these particles the four-momentum transfer squared $u$ is time-like for small $\Theta^*$, goes through zero, and becomes space-like for larger $\Theta^*$. At 3.5 GeV/c the change in sign occurs for $\cos \Theta^* = 0.959$, and 8 GeV/c for $\cos \Theta^* = 0.988$.

In Fig. 2, a, b have plotted the results of the experiments at BNL and CERN. Other experiments support these data, but they are not extensive enough to draw any definite conclusions.

The influence of scattering through resonance states is probably not important at 3.5 GeV/c, since total cross-sections and differential cross-sections at 180° vary rather smoothly in this neighbourhood.

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This point will be checked when the experiment at CERN\(^3\), which was done at several momenta in this region, has been completely analyzed.

We see from Fig. 2 that the \(\pi^+p\) scattering at 3.5 GeV/c can be fitted with an exponential function with a slope \(A = \frac{d}{du} \left( \ln \frac{d\sigma}{du} \right)_{u=0} = 11 \pm 3 \text{(GeV/c)}^{-2}\). At 8 GeV/c, \(A = 17 \pm 3\) for \(\pi^+p\) scattering\(^4\), while the data for \(\pi^-p\) are not accurate enough. All other experiments indicate a fast rise also for \(\pi^-p\) elastic scattering near 180°\(^5\).

The apparent shrinkage of the backward peak, at least in the case of \(\pi^+p\) scattering, make it suitable to parametrize the data according to reggeized particles\(^6\). Assuming for the moment that we are dealing with the exchange of a single Regge trajectory \(\alpha_E(V/u)\) in Fig. 1, we have

\[
\frac{d\sigma}{du} = \alpha_E(V/u) + \alpha_E(-V/u) - 2j(u). \tag{1}
\]

In Fig. 3 we have plotted \(\frac{d\sigma}{du}_{u=0}\) as a function of \(s\).

Other data exist\(^7\) which in general agree with the figure when extrapolated to \(u = 0\) with a slope given above. We find

\[
\left(\frac{d\sigma}{du}\right)_{u=0} = \text{const} x s^{-2}.
\]

Thus

\[
2[\alpha_E(0) - 1] = -3, \quad \alpha_E(0) = -0.50. \tag{2}
\]

The variation in the slope

\[
\frac{d(A)}{d(\ln s)} = \alpha_E''(0)
\]

where the differentiation is with respect to \(\sqrt{u}\). With the rate of decrease of \(A\) given above, we find

\[
\alpha_E''(0) = 1.1 \text{ (GeV/c)}^{-2}.
\]

This value lies within the range obtained for other Regge trajectories\(^8\).

If the description in terms of Regge trajectories bears any relationship to reality, one would expect several trajectories to contribute to \(\pi^+p\) backward scattering, while probably only one to \(\pi^-p\). We do not need this complication, at least not as yet.