Anomaly in the Energy Dependence of the Angular Distributions for Deuterons 
Scattered from Mg\(^{24}\) in the Energy Range from 6 to 13 MeV \(^{1}\)

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Dedicated to Professor Dr. W. GENTNER on the occasion of his 60th birthday

Angular distributions for elastic and 2\(^{+}\) inelastic deuteron scattering from Mg\(^{24}\) have been obtained at various energies in the range from 6 to 13 MeV. Yield functions have been measured in 50 keV steps at 90\(^{\circ}\) and 135\(^{\circ}\). In the region from 10 to 13 MeV, the elastic angular distributions show an anomalous behavior: instead of moving in from 180\(^{\circ}\), a new maximum is created by the breakup of a broad maximum at about 100\(^{\circ}\) into two maxima. The inelastic angular distributions do not show such an anomalous energy dependence. Therefore the Blair phase rule between the elastic and the 2\(^{+}\) inelastic angular distributions breaks down in the energy region of the anomaly in the elastic scattering, but it holds outside this energy region. In spite of fluctuations in the excitation functions, an optical-model analysis of the elastic angular distributions yields parameters which, with the exception of \(W\), vary smoothly with energy.

In previous work \(^{2,3}\) on elastic and inelastic deuteron scattering from Mg\(^{24}\), it was found that the analysis of the angular-distribution data in terms of diffraction-scattering theory \(^{3}\) showed an anomaly in the energy region from 9.5 up to about 14 MeV, the interaction radii for elastic and inelastic scattering were found to be different from each other, quite in contrast to theoretical expectations and to the results of the analysis of scattering data at higher energies (\(\geq 15\) MeV). It was shown \(^{2}\) that the \(R_0\) anomaly persists in the same way if the spatial extension of the deuteron is taken into account.

According to its limitations, the diffraction-scattering model was applied mainly to the more forward angular region of the differential cross section — i.e., just to the angular region investigated in most of the older cyclotron experiments, which were used for the \(R_0\) analysis. A complete list of references is given in ref. \(^{1}\). Unfortunately the older data were incomplete and suffered from relatively poor energy resolution and the fact that the targets were made from natural Mg instead of isotopically separated material.

To study the anomaly in more detail, we decided to measure a new, complete set of angular distributions in the energy range from 6 to 13 MeV, and over an angular range covering the backward hemisphere. In particular, the investigation was carried out to learn more about the energy dependence of the shape of the elastic and inelastic angular distributions and the validity of the Blair phase rule in deuteron scattering. It was hoped also that the experimental results would show whether or not the anomaly in \(R_0\) is connected with certain other general anomalies that have been reported for elastic deuteron scattering \(^{4}\).

Results and Discussion

The deuteron beam of the Argonne tandem Van de Graaff accelerator was used at various energies in the range from 6 to 13 MeV to measure angular distributions of elastically and 2\(^{+}\) inelastically scattered deuterons. The experiments were performed with an 18-inch scattering chamber and a \(dE-E\) solid state counter telescope. The target was highly enriched in Mg\(^{24}\) (\(\geq 98\%\) Mg\(^{24}\)) and had a thickness of about 1 mg/cm\(^{2}\). Absolute cross sections were obtained from the known distances and angles and the target thickness to an accuracy of approximately 10\%. The statistical errors were always less than 3\%.

\(^{2}\) J. S. Blair, Phys. Rev. 115, 928 [1959]. For a more complete list of references, see ref. \(^{1}\).
\(^{3}\) R. H. Siemssen, Argonne National Laboratory, Argonne, Ill., U.S.A.

Yield functions (Fig. 1) were measured in steps of 50 keV at 90° and 150°.

Fig. 2 shows the measured angular distributions for the elastic scattering (Fig. 2 a) and for the inelastic scattering, exciting the 1.37 MeV 2⁺ level of the ground state rotational band of Mg²⁴ (Fig. 2 b). The angular distributions show well developed diffraction patterns.

From diffraction scattering theories, it is expected that new maxima should move in from 180° as the energy is increased. However, Fig. 2 a shows the normal energy dependence only up to about 10 MeV; in this energy region the maxima and minima move towards smaller angles with increasing energy and at 9—10 MeV there is some indication that a new maximum is about to move in from 180°. Above 10 MeV, new maxima are created quite anomalously: the broad maximum in the 10 MeV angular distribution at about 100° and its straight long downward slope towards higher angles break up into two maxima as the energy is increased. These maxima persist then in the higher energy angular distributions at 11, 12.5, and 13 MeV. It seems that we have here a good example of a general anomaly in the elastic scattering of deuterons. It is interesting to note that the anomalous maxima occur in just the energy region of the $R_0$ anomaly.

A general feature of inelastic-scattering theories is the Blair phase rule, which states in our case...
that the oscillations in the elastic angular distribution should be out of phase with those of the $2^+$ inelastic angular distribution. Fig. 4a shows that the phase rule holds for the elastic and the $2^+$ inelastic angular distributions in the energy range from 7 to 10 MeV. Fig. 4b compares the shapes of elastic and $2^+$ inelastic distributions above 10 MeV incident deuteron energy, i.e., in the region of the anomaly. Apparently the Blair phase rule does not hold any more in the backward hemisphere of the angular distributions. Thus the phase rule breaks down just in the anomaly region where there is an extra maximum in the elastic angular distributions. It would be interesting to see whether such violations of the phase rule occur also in other cases in which the elastic angular distribution shows anomalies similar to those in our case.

It is clear that the violation of the Blair phase rule is mainly due to peculiarities in the elastic scattering. Therefore it is of special interest to see whether the anomaly in the elastic angular distributions can already be reproduced with the ordinary optical model. In view of the fluctuations in the differential yields (Fig. 1), however, it does not seem meaningful to try to fit the data in every detail. The best one can hope to obtain is a set of parameters that vary smoothly with energy and which then reproduce the main features of the measured angular distributions. Therefore, two series of calculations were performed. In the first, only data points up to a maximum angle of $110^\circ$ were considered, since compound elastic scattering will contribute most to the backward angles, where the cross sections for “direct” scattering are smallest. In the second group

Fig. 2. Angular distributions for (a) the elastic and (b) the $2^+$ inelastic deuteron scattering from Mg$^{24}$ at several bombarding energies in the range from 6 to 13 MeV. The ratio $\sigma/\sigma_{\text{Ruth.}}$ for elastic scattering and the differential cross section for inelastic scattering (both in arbitrary units) are plotted as functions of the c.m. scattering angle. The angular distributions are arbitrarily placed one above another to facilitate comparison of their shapes. A list of cross sections is available on request.
Fig. 3. Angular positions of minima and maxima in the angular distributions of elastic deuteron scattering from Mg\textsuperscript{24} as a function of energy.

The elastic-scattering data were analyzed with the optical-model search code JIB\textsuperscript{3} of PEREY\textsuperscript{5}. The potentials were taken to be of the standard Woods-Saxon type with a Saxon derivative form factor for the imaginary part, as defined by the expression

\[ U(r) = V_e(r) - V f(r, r_0, a) + 4 i a f' f(r, r_0, a), \]

with

\[ f(r, r_0, a) = \left[ 1 + \exp \frac{r-r_0 A^{1/5}}{a} \right]^{-1}. \]

Fig. 4. A comparison of elastic and 2\textsuperscript{+} inelastic angular distributions at several energies (a) in the range from 7 to 10 MeV and (b) in the range from 11 to 13 MeV. The elastic cross sections are given as \( \sigma/\sigma_{\text{Roth}} \) on an arbitrary scale. The elastic angular distributions are displaced vertically to facilitate comparison with the inelastic ones, which are plotted on the same arbitrary scale. The Blair phase rule is seen to hold in the range from 7 to 10 MeV, but it breaks down at the higher energies in the backward hemisphere.

No spin-orbit coupling was taken into account. The COULOMB potential \( V_c \) was that of a uniformly charged sphere of radius \( R_c = 1.2 \, \text{Å}^{1/3} \) fm.

Very similar optical-model parameters were obtained from the search calculations that included all data points and from the calculations restricted to forward angles. In Fig. 5 the data are compared with the theoretical angular distributions obtained (a) from the search restricted to data points at angles \(< 110^\circ\) (dotted curves) and (b) from the search that included all data points (solid curves). The optical-model parameters from the search on all data points are listed in Table 1 and are graphically displayed in Fig. 6. No satisfactory fits have been obtained for the 6 and 12.5 MeV data. Therefore these results have not been included in Fig. 6.

Table 1. Optical model parameters for deuterons scattered from Mg\(^{24}\) (\( r_c = 1.2 \, \text{fm}\)).

<table>
<thead>
<tr>
<th>( E_d ) (MeV)</th>
<th>( V ) (MeV)</th>
<th>( r_0 ) (fm)</th>
<th>( a ) (fm)</th>
<th>( W ) (MeV)</th>
<th>( r_0' ) (fm)</th>
<th>( a' ) (fm)</th>
</tr>
</thead>
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<tr>
<td>7.0</td>
<td>72.5</td>
<td>1.05</td>
<td>1.022</td>
<td>23.3</td>
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<td>.471</td>
</tr>
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<td>8.0</td>
<td>74.8</td>
<td>1.05</td>
<td>.969</td>
<td>17.0</td>
<td>1.60</td>
<td>.597</td>
</tr>
<tr>
<td>9.0</td>
<td>79.0</td>
<td>1.05</td>
<td>.893</td>
<td>17.5</td>
<td>1.45</td>
<td>.674</td>
</tr>
<tr>
<td>10.0</td>
<td>81.0</td>
<td>1.05</td>
<td>.804</td>
<td>17.3</td>
<td>1.35</td>
<td>.734</td>
</tr>
<tr>
<td>11.0</td>
<td>87.4</td>
<td>1.05</td>
<td>.820</td>
<td>22.8</td>
<td>1.34</td>
<td>.680</td>
</tr>
<tr>
<td>13.0</td>
<td>77.7</td>
<td>1.05</td>
<td>.802</td>
<td>21.3</td>
<td>1.28</td>
<td>.701</td>
</tr>
</tbody>
</table>

The optical-model parameters are found to vary smoothly with bombarding energy, with the exception of the imaginary potential \( W \) which is considerably larger in the anomaly region at 11 and 13 MeV than in the energy region from 8 to 10 MeV.

From Fig. 5 it is seen that the theoretical curves reproduce the main features of the experimental angular distributions remarkably well in spite of the fluctuations in the yield functions. The smooth and systematic variation of the parameters is consistent with the optical model. The theoretical curve in Fig. 5 for the 12.5 MeV scattering has been calculated with parameters averaged from those at 11 and 13 MeV.

Calculations with average geometrical parameters \( a, a', \) and \( r' \) showed that the anomaly could not be
reproduced. The variation of these parameters with energy (Fig. 6) is needed to reproduce the splitting of the maximum at 110° at energies above 10 MeV. Of special interest is the increase in the imaginary radius as the bombarding energy is decreased. A similar increase in the imaginary radius was observed by Bassel et al. in an analysis of the scattering of deuterons from Ca\textsuperscript{40}. These authors offer a tentative explanation for the increase of the imaginary radius with decreasing bombarding energy. If a large fraction of the absorptive potential stems from direct reactions and these are localized at the nuclear surface, then the Coulomb repulsion causes the reaction zone to move outward as the bombarding energy decreases. If this explanation is correct, this effect is expected to be larger for nuclei with larger $Z$. However, this conclusion seems not to be supported by our results.

We are indebted to Dr. F. Perey for making his optical-model search code JIB3 available to us. The help of J. McShane in the data acquisition is gratefully acknowledged.

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