Doorway States and Intermediate Structure Phenomena

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(Dedicated to Professor Dr. W. Gentner on the occasion of his 60th birthday)

The phenomenon of the distribution of a doorway state over many complicated compound nuclear states, i.e., the formation and properties of a so-called micro-giant resonance, are investigated analytically and numerically by means of a simple model. The distribution of the poles in the complex plane and the distribution of the residues of the S-matrix are obtained numerically and used to calculate the average cross-section. It is shown that the Lane-Thomas-Wigner theory applies, except for a possible correction to the shape of the resonance occurring in the cross-section averaged over the energy. This shape need not be describable by a Lorentzian. Estimates of the "spreading width" \( W \) and the "total width" \( W_{0} \) are given for particular cases. It is noteworthy that the relationship \( W_{0} = W + \Gamma \) need not be correct even as an order-of-magnitude estimate, where \( \Gamma \) is the decay width of the doorway state.

Doorway states resulting from the two-body character of the nuclear forces and functioning as intermediaries between the continuum and the complicated compound nuclear states should manifest themselves through intermediate structure phenomena in nuclear excitation functions. Recently, indications have been mounting for the existence of such phenomena, at least in certain regions of the periodic table. The isobaric analogue states, too, are manifestations of this phenomenon. In the present paper, we analyze the formal properties of doorway states and, in particular, investigate the intermediate structure phenomena induced by such states. This intermediate structure depends upon the various coupling constants (matrix elements) which enter into the theory.

A doorway state is defined as a bound state of a model-Hamiltonian embedded in the continuous spectrum with the property that the matrix element of the complete Hamiltonian operator between \( \Phi \) and the continuum is not small. Clearly, this definition is dependent both on the model-Hamiltonian and on the channel (the continuum) under consideration. In order for intermediate structure phenomena to exist, it is necessary that the density of doorway states be sufficiently small. Otherwise, the doorway states overlap and intermediate structure connected with individual doorway states gets lost. This condition (that the density of doorway states be small) is met in the light nuclei where we deal with isolated resonances for sufficiently low energies. In the medium-weight and heavy nuclei, intermediate structure phenomena associated with individual doorway states are expected to show up most strongly in reactions involving simple projectiles (nucleons) and target nuclei in the vicinity of closed shells. Also, such phenomena may appear favourably in particular ranges of excitation energies of the compound nucleus. However, it is possible that collective excitations may also function as doorway states. At any rate, it is clear that even isolated doorway states in medium-weight and heavy nuclei will in most cases not appear as individual resonances, but instead as "microgiant" resonances and it is this case that we are presently interested in.

5. H. Feshbach, Rev. Mod. Phys. 36, 1076 [1964].
isolated doorway state is “smeared out” over a large number of compound nuclear states. Thereby, the partial widths of these states relating to particular channels acquire coherence properties that lead to a microgiant resonance. The existence of such coherence properties is, in fact, a model-independent definition of a doorway state. In the following, we investigate this coherence phenomenon analytically and numerically. We are particularly interested in the increase of the “smearing” with increasing coupling between doorway state and the complicated compound states, and in the correlations in magnitude and sign of the residues of the poles of the S-matrix. The precise knowledge of these correlations is necessary for the calculation of the average cross-section, i.e. for the understanding of the intermediate structure phenomenon.

In sect. I, we define our assumptions and notation. We describe general properties of the doorway state which can be obtained analytically. In sect. II, we describe the calculation of average cross sections and the dependence of the resulting parameters on the input parameters of the theory. In sect. III, we give some of the results of extensive numerical calculations.

I. A Model for a Doorway State

We confine ourselves to one channel (elastic scattering only). The formalism employed here is similar to that of refs. 10 and 15 and will not be explained unless it differs from these references. Given a model HAMILTONIAN $H_0$ with one doorway state $\Phi$ and a large number of complicated states $\Phi^{(i)}$, $i = 1, \ldots, N$ embedded in a continuum with scattering functions $\psi_E$, we assume that the complete HAMILTONIAN $H$ has the following real matrix elements

$$
\langle \psi_E | H | \psi_{E'} \rangle = E \delta(E - E'),
$$

$$
\langle \psi_E | H | \Phi \rangle = V_E,
$$

$$
\langle \psi_E | H | \Phi^{(i)} \rangle = 0,
$$

$$
\langle \Phi | H | \Phi \rangle = \varepsilon_i,
$$

$$
\langle \Phi | H | \Phi^{(i)} \rangle = V_i,
$$

$$
\langle \Phi^{(i)} | H | \Phi^{(j)} \rangle = \varepsilon_{ij} \delta_{ij}.
$$

We thus assume that $H$ has been diagonalized in the subspace of functions $\Phi^{(i)}$ and $\psi_E$. We also assume that the elastic scattering phase shift $\delta$


obtained from the asymptotic behaviour of the functions $\psi_E$ and defined in equ. (3.3) of ref. 10 is constant over the energy range of interest, and that consequently $V_E$ can be approximated by a constant, $V$ say.

The S-matrix (now called $S$-function as we deal with a one-channel problem) has $N + 1$ poles, corresponding to the coupling of the $N$ states $\Phi^{(i)}$ and the state $\Phi$ to the continuum. As shown in refs. 10 and 15, it has the general form

$$
S = \exp(2i\delta) \cdot \left(1 - i \sum_{a=1}^{N+1} \frac{\Gamma_a}{E - \mu_a}\right)
$$

(2)

with

$$
\sum_{a=1}^{N+1} \Gamma_a = 2\pi V^2; \sum_{a=1}^{N+1} \mu_a = \sum_{i=1}^{N} \varepsilon_i + \varepsilon - i\pi V^2.
$$

(3)

The complex energies $\mu_a$ are defined as the roots of the equation

$$
D(E) = \begin{bmatrix}
E - \varepsilon_1 & 0 & -V_1 \\
E - \varepsilon_2 & \ldots & -V_2 \\
0 & \ldots & 0 \\
-V_1 & -V_2 & \ldots & -V_N & E - \varepsilon_N & -V_N
\end{bmatrix} = 0.
$$

(4)

This equ. can be transformed into

$$
\prod_{i=1}^{N+1} (E - \varepsilon_i) = \prod_{i=1}^{N} V_i^2 \prod_{i=1}^{N} (E - \varepsilon_i)
$$

(5)

with $\varepsilon_{N+1} = \varepsilon - i\pi V^2$. From equ. (2.6) of ref 15, it follows that

$$
\Gamma_a = 2\pi V^2 \prod_{i=1}^{N} \frac{\mu_a - \varepsilon_i}{\prod_{\beta=1}^{N+1} \frac{\mu_a - \varepsilon_\beta}{(\mu_a - \varepsilon_\beta)^2}}.
$$

(6)

By taking the derivative of $D(E)$ with respect to $E$ at $E = \mu_a$ and using $D(E) = \prod_{a=1}^{N+1} (E - \mu_a)$ and equ. (5), it can be shown that

$$
\Gamma_a = \frac{2\pi V^2}{1 + \sum_{i=1}^{N} \frac{V_i^2}{(\mu_a - \varepsilon_i)^2}}.
$$

(7)

Then, $\Gamma_a$ can simply be calculated once the roots $\mu_a$ of equ.(5) are known. Similarly, it is possible to show that

$$
\text{Im} \mu_a = \frac{-\pi V^2}{1 + \sum_{i=1}^{N} \frac{V_i^2}{(\mu_a - \varepsilon_i)^2}}.
$$

(8)
so that \( \lim_{\omega \to 0} (\Omega_\omega - 2 \text{Im} \mu_\omega) = 1 \) as should be the case.

The equs. given above can be interpreted physically as follows. Let us first discuss the case where \( V_i = 0 \) for all \( i \). Then, \( \mu_i = \epsilon_i \) for \( i = 1, \ldots, N+1 \), and \( \Gamma_i = 0 \) for \( i \leq N \). Furthermore, \( \Gamma_{N+1} = 2 \pi V^2 \).

The \( S \)-function has a single pole at a distance \( \frac{1}{2} \Gamma \) from the real axis, corresponding to the doorway state. This pole manifests itself in the elastic scattering cross-section \( \sigma(E) \) through the occurrence of an isolated resonance. Here, \( \sigma(E) \) is (except for factors not interesting here) given by

\[
\sigma(E) = \left| S(E) - 1 \right|^2 = 2 \left( 1 - \text{Re} S(E) \right).
\]  

As we turn on the couplings \( V_i \), the function \( S(E) \) acquires \( N+1 \) poles, which according to equ. (8) are all situated below the real axis. The doorway-state pole moves toward the real axis. This follows from the second of equs. (3). At the same time, the strength \( 2 \pi V^2 \) with which the doorway state \( \Phi \) is coupled to the continuum is spread over the \( N+1 \) resonances which now arise, according to the first of equs. (9). As long as the matrix elements \( V_i \) are sufficiently small, we may use perturbation theory to calculate \( \mu_a \) and \( \Gamma_a \) and obtain

\[
\mu_k = \epsilon_k + \frac{V_k^2}{\epsilon_k - \epsilon_{N+1}}; \quad k = 1, \ldots, N;
\]

\[
\mu_{N+1} = \epsilon_{N+1} + \sum_{k=1}^{N} \frac{V_k^2}{\epsilon_{N+1} - \epsilon_k}, \quad (10)
\]

and

\[
\Gamma_k = 2 \pi V^2 \frac{V_k^2}{(\epsilon_k - \epsilon_{N+1})^2}; \quad k = 1, \ldots, N;
\]

\[
\Gamma_{N+1} = 2 \pi V^2 - \sum_{k=1}^{N} \Gamma_k. \quad (11)
\]

In this case, the spreading of the doorway state over the compound states is very small. We call this situation case a. It is remarkable to see that the perturbation-theoretical denominators occurring in equs. (10) and (11) contain both the positions and the widths of the unperturbed states. This shows that the quantity of interest for a perturbation-theoretical approach is not the difference of (real) energies between the states (this might become arbitrarily small for sufficiently high densities of the states \( \Phi^{(1)} i \)), but the difference between the poles in the complex plane. The pattern resulting in the scattering amplitude for case a is that of a single pole providing the background (a broad resonance), superimposed upon which the \( N \) poles of the states \( \Phi^{(1)} i \) produce quick variations. In this case, we do not encounter a doorway state in the true meaning of the word: The state \( \Phi \) decays back into the continuum long before it starts populating the states \( \Phi^{(1)} i \), so that, although it fulfills the formal definition of a doorway state, it actually blocks the entrance to the compound nucleus.

As we further increase the strength of the matrix elements \( V_i \), the doorway state pole approaches the \( N \) other poles corresponding to the states \( \Phi^{(1)} i \), until it merges with them. Thereby, the doorway state \( \Phi \) losess its identity and becomes spread over the \( N+1 \) compound nuclear resonances. We now consider a situation extremely opposite to case a, where the matrix elements \( V_i \) are large, and where the merger and the distribution of \( \Phi \) over the \( N+1 \) resonances are so complete that each of the \( N+1 \) quantities \( \mu_a \) has a very small imaginary part that can be treated in perturbation theory. In that case, the matrix occurring in equ. (4) without the term \( i V^2 \) in the \( (N+1, N+1) \) element can be diagonalized by a real orthogonal matrix \( O_{ik} \) with resulting real eigenvalues \( \lambda_i \). The elements \( O^*_{N+1} \) give the distribution of the doorway state over the \( N+1 \) eigenstates of the system; they will be assumed to have a Lorentzian distribution,

\[
O^2_{N+1} = \frac{W d}{2 \pi [(\lambda_i - \epsilon)^2 + \frac{i}{2} W^2]}, \quad i = 1, \ldots, N+1, \quad (12)
\]

where \( W \) is the "spreading width" of the doorway state over the eigenstates, and \( d \) is the average distance between neighbouring eigenvalues \( \lambda_i \). The quantity \( W \) has been estimated in refs. 4 and 16, these authors find

\[
W \approx \frac{2 \pi d}{V^2} \quad (13)
\]

where \( V^2 \) is the average of \( V_i^2 \). Transforming the matrix in equ. (4) with \( O_{ik} \), we obtain the new determinant

\[
D(E) = \left| (E - \lambda_j) \delta_{jk} + \frac{i}{2} O_{jN+1} O_{k N+1} \Gamma \right| = 0. \quad (14)
\]

If \( W \gg \Gamma = 2 \pi V^2 \), perturbation theory in equ. (14) becomes, according to equ. (12), applicable and we obtain for the eigenvalues \( \mu_a \) the expression

\[
\mu_j = \lambda_j - \frac{i}{2} O^2_{jN+1} \Gamma; \quad j = 1, \ldots, N+1. \quad (15)
\]

Correspondingly, we have in lowest order in \( \Gamma/W \)

\[
\Gamma_j = O^2_{jN+1} \Gamma; \quad j = 1, \ldots, N+1. \quad (16)
\]

The case $W \gg I$, henceforth called case b, does lead to a true doorway-state phenomenon: While the population of the $N+1$ compound states is possible only through the doorway state $\Phi$, each of these $N+1$ states is populated, and the original doorway state has totally disappeared. It is smeared out over the compound states and manifests itself only in a coherence property of the widths $\Gamma_a$, as shown by equs. (16) and (12). Unfortunately, case b seems as unrealistic as case a. Any intermediate case can only be treated numerically which is done in sect. III.

II. Average Cross Sections

The average cross section $\langle \sigma(E') \rangle_E$ is defined by

$$\langle \sigma(E') \rangle_E = \int \frac{dE'}{E-E'} \sigma(E') f(E').$$

(17)

According to ref. 17, we choose

$$f(E') = \frac{1}{\pi} \left[ (E'-E)^2 + I^2 \right]^{-1}$$

(18)

and take $L \to \infty$. Using the second of equs. (9), and eqn. (2) we obtain 17

$$\langle \sigma(E') \rangle_E = (\cos 2 \delta) \cdot \left( \sum_{\alpha=1}^{N+1} \frac{i \Gamma_a}{E - \mu_a + i I} + \text{c.c.} \right) + (\sin 2 \delta) \cdot \left( \sum_{\alpha=1}^{N+1} \frac{\Gamma_a}{E - \mu_a + i I} + \text{c.c.} \right) + 2(1 - \cos 2 \delta).$$

(19)

We calculate the expression (19) for cases a and b. We make the usual approximations, i.e. $d \ll 1$ where for case b the quantity $d$ is defined as the average distance of the $\text{Re} \mu_a$. Then, the sums occurring in eqn. (19) can be transformed into integrals. In case a we furthermore assume that $|\text{Im} \mu_a| \ll 1$ for $\alpha \neq N+1$, that $\Gamma' \gg 1$ and that $V_i^2 = \alpha \ll \Gamma$ with an independent of $i$. Under these approximations, we obtain

$$\sum_{\alpha=1}^{N+1} \frac{\Gamma_a}{E - \mu_a + i I} \rightarrow \frac{1}{E - \varepsilon + \frac{1}{2} i W}.$$  

(20 a)

In other words, the average cross section obtained from equs. (19) and (20) displays the single doorway-state resonance, while the superimposed oscillations have disappeared. For case a this should be expected. For case b, we use the assumption $W \gg I \gg |\text{Im} \mu_a|$ and obtain

$$\sum_{\alpha=1}^{N+1} \frac{\Gamma_a}{E - \mu_a + i I} \rightarrow \frac{1}{E - \varepsilon + \frac{1}{2} i W}.$$  

(20 b)

Again, because of $I \ll W$ this ought to be expected, but yields a different expression for the average cross section than that obtained from eqn. (20 a). The case intermediate between a and b cannot be treated analytically unless several assumptions are introduced. We postpone a justification of these assumptions to the next section. We do not now assume that $W \gg \Gamma$. However, we are interested in a case where the doorway state pole has already merged with the other $N$ poles. In this case, the quantities $\Gamma_a$ are no longer real. For simplicity, we assume that the quantities $V_i^2$ are distributed symmetrically, i.e. that $V_i^2 = V_{N+1}^2$. It then follows from eqn. (5) that the $\mu_a$'s are distributed symmetrically about $\varepsilon$, and from eqn. (7) that the quantities $\text{Re} \Gamma_a$ are distributed symmetrically, the quantities $\text{Im} \Gamma_a$ skew-symmetrically about $\varepsilon$. For $I \gg d$, we can again replace summations by integrals and therefore deal with a function $\Gamma(E)$ obtained from the quantities $\Gamma_a$. We describe $\Gamma(E)$ by the formula

$$\Gamma(E) = \frac{1}{2 \pi} \int \frac{1}{(E - \varepsilon + \frac{1}{2} i W)^2} dE,$$

(21)

which is in accord with the first of equs. (3). Under the further assumption $|\text{Im} \mu_a| \ll 1$ we can calculate the average cross section, we obtain for $I \ll W_0, W_1$

$$\langle \sigma(E') \rangle_E = \frac{\Gamma_1 W_1}{(E - \varepsilon + \frac{1}{2} i W)^2} - i \frac{\Gamma_1 W_1}{(E - \varepsilon + \frac{1}{2} i W)^2}.$$  

(22)

In order to discuss qualitatively the change resulting from eqn. (22) in the average cross section, we first write the average cross section for case b, combining equs. (19) and (20 b)

$$\langle \sigma(E') \rangle_E = \frac{\Gamma W}{(E - \varepsilon + \frac{1}{2} i W)^2} \cos 2 \delta$$

$$+ \frac{2 \Gamma (E - \varepsilon)}{(E - \varepsilon + \frac{1}{2} i W)^2} \sin 2 \delta + 2(1 - \cos 2 \delta).$$  

(23)

We now assume $W_0 \approx W_1$ and define $z = \Gamma_1/\Gamma$. Then, we obtain from eqn. (22) a formula similar to eqn. (23), except for the following changes. The quantity $W$ in eqn. (23) has to be replaced by $W_0$.
The term in front of \( \cos 2 \delta \) has to be multiplied by

\[
A(E) = (1 + 2x) \frac{(E - \varepsilon)^2 + \frac{1}{2} W_0^2 (1 - 2x)}{(E - \varepsilon)^2 + \frac{1}{2} W_0^2 (1 + 2x)},
\]

(24 a)

the term in front of \( \sin 2 \delta \) by

\[
B(E) = \frac{(E - \varepsilon)^2 + \frac{1}{2} W_0^2 (1 - 4x)}{(E - \varepsilon)^2 + \frac{1}{2} W_0^2 (1 + 4x)} \]

(24 b)

It can easily be seen that both \( A(E) \) and \( B(E) \) lead to an effective widening (narrowing) of the resonance for \( x > 0 \) \((x < 0)\). This statement is of interest because from the measured shape of an intermediate structure resonance, one would like to extract information regarding the quantities \( T \) and \( W \), the latter being defined in equ. (13). The questions to be answered by the numerical calculations can thus be summarized as follows: I) For which values of \( W \) does the merger between the doorway state and the complicated states take place? II) Is the description (21) for the distribution of the \( T_{\beta} \)'s realistic? III) What are the values for the quantities \( W_0, W_1, \Gamma_1 \) in dependence on \( \Gamma \) and \( W \)? Can we put \( W_0 \approx W_1 \), and is it reasonable to write \( \mu \approx \Gamma + W? \)

### III. Numerical results

We have investigated numerically the solutions of the equs. (5) and (7), using the following input data. We put \( \varepsilon_{N+1} = 25.5 - i \beta \) and \( N = 50 \), so that \( \varepsilon_j = j, \; j = 1, \ldots, 50 \). All matrix elements \( V_1^2 \) were assumed to be equal, \( V_1^2 = 2 \). The solutions thus depend upon the two parameters \( \alpha \) and \( \beta \). In order to have \( N \gg \beta \gg d \approx 1 \), we have chosen \( \beta = 8 \), which corresponds to a width \( \Gamma = 16 \). We have also investigated the general behaviour of the solutions for different values of \( \beta \), but those obtained for \( \beta = 8 \) are typical so that we restrict our discussion to them. The quantity \( W \) defined by equ. (13) is given by \( A \approx 2 \pi \alpha \), since \( d \approx 1 \) with our choice of parameters. The merger between the doorway state and the many complicated states proceeds as follows:

For \( \alpha = 1 \), \( W \approx 6.28 \) the doorway state is still distinct from the complicated states, it is situated at a distance of 5.3 from the real axis, whereas the neighbouring poles have distances of about 0.2 from the real axis. For \( \alpha = 2 \), \( W \approx 12.57 \) the situation has not changed qualitatively, the corresponding figures are now 2.0 and 0.3, respectively.

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Table 1. Distribution of the poles \( \mu_\alpha \) and residues \( \Gamma_\alpha/\Gamma \) of the S-function.

For \( \alpha = 3 \), we have \( W \approx 18.85 \), so that \( W \approx \Gamma = 16.0 \), and the merger is complete. Table 1 shows in the first and second column the distribution of the real and imaginary parts of the quantities \( \mu_\alpha, \beta = 1, \ldots, 51 \) [see equs. (2) to (4)] for \( \alpha = 3 \), and in the third
and fourth column the values of the real and imaginary parts of the quantities $T^{\beta}/\Gamma$ [see equ. (7)] for $\beta = 1, \ldots, 51$ and $\alpha = 3$. This result (as well as the other cases investigated numerically) shows that the merger between the doorway state pole and the other poles becomes complete at values of $W \cong \Gamma$.

We now turn to the distribution of the quantities $T^{\beta}/\Gamma$ (Table 1, third and fourth column). In the first place, it is interesting to notice that $T^{\beta}/\Gamma$ is not related to $\text{Im} \mu^{\beta}$ in any simple way. This should be expected since we deal with overlapping resonances. A comparison between column three of that table and equ. (21) shows that is not possible to fit the distribution exactly by a Lorentzian. This is because according to equ. (21) the real part of $T^{\beta}/\Gamma$ should be given by

$$d^{W_{\beta}}_{\frac{1}{2}} \frac{1}{\pi (E^{\beta} - \epsilon)^{2} + \frac{1}{4} W_{\beta}^{2}}.$$ 

This expression contains only one parameter, $W_{\beta}$, which is determined by the height of the distribution at $E^{\beta} = \epsilon = 25.5$. We obtain $W_{\beta} = 6.7$, and with this value $T^{\beta}/\Gamma = 0.031$ at $E = 31.266$, whereas the value from Table 1 is 0.021. On the other hand, at $E = 50.180$ we obtain $T^{\beta}/\Gamma = 0.0017$, in contrast to the value 0.0071 from Table 1. Probably part of this discrepancy is due to the fact that we have used a finite range of energies, whereas equ. (21) applies to an infinitely large energy interval. It is clear, however, that a Lorentzian distribution for $\text{Re} T^{\beta}/\Gamma$ applies only rather approximately, in particular since it also depends upon the distribution of the quantities $V^{\beta}$. A better fit to the wing of the distribution is obtained by decreasing $W_{\beta}$; then, however, the values obtained near the center of the distribution are too small. The quantity $W_{\beta}$ may be called the spreading width, it is extremely interesting that the relationship $W_{\beta} = \Gamma + W$ applies not even as an order-of-magnitude estimate, since $W_{\beta} = 3 \ldots 7$ is the total width entering into the formula for the averaged cross-section, equ. (23), and $\Gamma \cong W \cong 16 \ldots 18$.

The distribution of the imaginary part of $T^{\beta}/\Gamma$ (column four of Table 1) can be fitted more easily with the formula (21), simply because we now have two parameters at our disposal, the width $W_{1}$ and the height $I_{1}$ of the distribution. A rough fit to the distribution (done by hand) yields $W_{1} = 3.3$, which in view of the bad fit obtained for $\text{Re} T^{\beta}/\Gamma$ agrees rather well with $W_{\beta} = 6.7$, and $I_{1}/\Gamma = -0.045$. This shows that the quantity $x = T^{\beta}/I$ appearing in equ. (24) is exceedingly small, in spite of the fact that $\text{Im} T^{\beta}$ is of the same order of magnitude as $\text{Re} T^{\beta}$.

In addition, we wish to point out that the distribution of the quantities $\mu_{\alpha}$ was not in all cases as smooth as that shown in Table 1. If one increases the coupling $\alpha$ further and considers, f.i., a value $\alpha = 8.0$, one finds that the poles $\mu_{\beta}$ tend to repel each other and lie no longer on a smooth curve. Whether or not this effect was caused by the finite energy interval chosen could not be decided within the frame of the present investigation.

The calculations, carried out at the Deutsches Rechenzentrum in Darmstadt, were supported by the Deutsche Forschungsgemeinschaft. The author would also like to thank Dipl.-Ing. G. Engeli for programming the numerical calculations.