Time and Space-Resolved Electron Densities in a Theta Pinch at Various Distances along the Discharge Axis

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The influence of the finite length of the discharge coil on the axial behaviour of a theta pinch plasma column is investigated by measuring electron density profiles at various distances from the midplane of the coil along the discharge axis. The resulting line densities were then checked by an independent method. For a parallel trapped magnetic field it was possible in this way to measure end losses and also the collection rate of the contraction wave in the case of an antiparallel trapped magnetic field.

In previous investigations time-resolved measurements of the radial electron density distribution were carried out on a 26 kJoule theta pinch experiment. The results were compared with theoretical predictions of the three-fluid model for the behaviour of the plasma during the discharge. The agreement between the experimental and theoretical results was found to be very reasonable as long as the comparison was limited to the admissible time interval, i.e. as long as the plasma was undisturbed as in an infinitely long coil. It was also found, however, that for later times of the discharge discrepancies, some of them appreciable, occurred between the theoretical predictions and the measurements even before the peak magnetic field was reached. These were attributed to effects due to the finite length of the coil and were not further considered at the time. To make up for this, measurements of electron densities at various distances along the discharge axis were carried out in order to investigate the influence of the finite length of the coil on the plasma. An account of these measurements is given in the following. It was hoped to obtain information in this way on the quantitative amount of end losses of the plasma, especially in the case of a parallel trapped magnetic field and the collection rate by axial contraction with antiparallel trapped magnetic field.

Since these investigations were thus meant as a continuation of the earlier measurements the same discharge conditions were retained. Hence, the main capacitor bank of the 26 kJoule experiment, which had a quarter period of 1.6 μs, was fired after the hydrogen in the quartz discharge tube (inner diameter: 4.5 cm) had been preionized up to about 50% and a magnetic field of variable sign and magnitude had been superimposed.

The investigations were carried out for the following initial conditions at the moment when the main discharge was started:

1. Hydrogen filling pressure \( p(H_2) = 0.15 \text{ torr}, B_{zo} = 0.8 \text{ kG} \)

2. Hydrogen filling pressure \( p(H_2) = 0.10 \text{ torr}, B_{zo} = -2.3 \text{ kG} \)

The radial electron density distributions were again obtained from continuum measurements at \( \lambda = 4980 \text{ Å} \) by means of a light-pipe assembly, which picked up the plasma radiation from eight different distances from the discharge axis. In addition, the so-called line density \( \sigma \text{[cm}^{-1}\text{]} \) was derived from the duration of the individual radial plasma oscillations.

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as will be outlined in this report. The line density is defined as the number of electrons or ions in the whole plasma cross section with a tube axis 1 cm long.

\[ A) \quad B_{\theta 0} = 0.8 \, \text{kG}, \quad p(H_2) = 0.15 \, \text{torr} \]

Given first are the results of those measurements for which the trapped magnetic field in the plasma was weak and parallel to the driving one. They are very easily explained by smear camera pictures, taken in the following two ways. In the first the plasma radiation was observed through a slit in the discharge coil perpendicular to its axis and the image stigmatically focussed on the recording photographic plate was swept perpendicularly to the plasma diameter. This technique is illustrated in Fig. 1a, which gives an example of smear photos obtained in this way. Typical theta pinch radial plasma oscillations are shown, and in such pictures up to 13 compressions of the plasma could be identified in the case of a parallel trapped magnetic field. The second method of smear-camera recording is demonstrated in Fig. 1b. Here the plasma was observed through a coil slit parallel to the discharge axis. It consisted of two close lines of holes in the coil metal. It was thus possible to smear the axial behaviour of the luminous phenomenon inside the coil and also partly outside as illustrated by the example given. The first successive maximum compressions of the plasma are recognizable here as bright vertical lines. They are slightly bent, indicating that the plasma oscillates faster in the neighbourhood of the ends of the coil than about its mid-plane. It will be shown later on that this can be attributed to end losses during the applied preionization. It can also be seen from the smear picture in Fig. 1b that a luminous front moves into the tube volume outside the discharge coil with a velocity of about \(1 \times 10^7\) cm/s even before the first maximum compression of the plasma at \(t = 0.36\ \mu s\) occurs. This front can easily be explained as consisting of plasma which is driven out of the coil ends at the beginning of the main discharge by the magnetic field lines which diverge here. In Fig. 2, which is given later on, this plasma wave can be recognized again. Behind this first luminous front a continued radiation is recorded on the smear pictures. A continuous escape of plasma from the interior of the coil is responsible for this radiation.

In order to measure the plasma losses through the coil ends connected with this outflow, the line density:

\[
\sigma = 2\pi \int N_e \, r \, dr \\
R = 2.3 \, \text{cm} \\
r = 0.0 \, \text{cm}
\]

at different distances along the coil axis will be considered. This quantity can be gained from the continuum radiation with the eight light-pipe assembly. It was measured at four different distances from the midplane of the coil, namely at 2.9 cm, 7.9 cm, 12.9 cm and 16.0 cm from it. The coil end was at 15.0 cm in this scale. Reflection of plasma light at the inner coil and tube walls resulted, however, in not only the continuum radiation from the plasma column being recorded itself, but also in a considerable background intensity being superimposed on the radial intensity profiles. From the measurements this background could be established as fairly constant across the whole inner diameter of the tube. It was, therefore, easily subtracted from the recorded continuum intensity profiles, as its magnitude at each moment could be derived from the measurement itself.

On the other hand, this supplementary correction made it necessary to check the resulting line densities by another independent measurement. For this purpose the time differences \(\tau\) between the individual compressions of the plasma were taken. They were obtained from the continuum measurements themselves as well as from the smear pictures, in which, as already mentioned, up to 13 maximum compressions could be identified, as suggested, for example, by Fig. 1a.
The relation between these time differences \( \tau(t) \) and the line density \( \sigma(t) \) can easily be derived:

\[
\sigma(t) = \frac{B_o^2(t)}{m} \cdot \left( \frac{\tau(t)}{2\pi} \right)^2
\]

\((B_o(t) = \text{external magnetic field, } m = \text{mass of the protons})\)

by proceeding from the most simple model for the plasma oscillations i.e. from a plasma ring at \( r_s \) with a very thin sheath thickness:

\[
m \sigma r_s = \frac{B_1^2 - B_2^2}{8\pi} \cdot 2\pi r_s,
\]

\[
\tau_s = \frac{(B_{10} r_0^2)^2}{4m \sigma} \cdot r_s^3 - \frac{B_o^2(t)}{4m \sigma} \cdot r_s
\]

and by assuming constancy for the flux of the internal magnetic field \((B_1 = B_{10} \cdot r_0^2/r_s^2)\) and also for the magnitude of the external magnetic field during a single radial oscillation. At the different moments this very simplified model already yields plasma radii in fairly good agreement with the experimental ones as long as the relatively slow damping of the real plasma oscillations may be neglected. The time differences \( \tau(t_1) \) and \( \tau(t_2) \) between the first two plasma oscillations, however, afforded too small values in this simplified model, whereas they were in agreement with the experimental values for a number of subsequent oscillations when the plasma close to the midplane of the coil was considered. The line densities \( \sigma(t_1) \) and \( \sigma(t_2) \) derived from \( \tau(t_1) \) and \( \tau(t_2) \) are not considered, therefore, in the following. Special reference should be made to the quadratic dependency of \( \sigma \) on \( \tau \), which causes considerable errors in the line densities due to an inaccurate evaluation of the moments of the maximum plasma compressions. The line densities derived from the plasma oscillations will, therefore, only be considered as a check on the absolute magnitudes of the corrected \( \sigma \)-values determined from the continuum measurements.

In Fig. 2 the results of the measurements are given for the case of a weak magnetic field trapped parallel in the plasma. The radial electron density

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**Fig. 2.** Experimental results of line densities and electron density profiles for a parallel trapped magnetic field at four different distances from the midplane of the discharge coil.
profiles at two selected moments \( t = 0.36 \mu s \), i.e. first maximum compression, and \( t = 1.00 \mu s \) are plotted as well as the time history of the line densities at three different distances along the discharge axis inside the coil and 1 cm outside it. The line density \( \sigma_0 \), which corresponds to the filling pressure \( p(H_2) = 0.15 \) torr, is also noted. It is apparent from Fig. 2 that in the experiment this value \( \sigma_0 \) is only attained in the interior of the coil, about the midplane. The measured line densities decrease towards the coil ends, as already suggested by the smear picture in Fig. 1b. This diminution, which was also observed in similar discharges by other authors, is attributed to plasma losses during the preionization. The next statement which can be made from Fig. 2 is that there is reasonable agreement between the results obtained by the two different methods of line density determination.

Considering first of all the time history of \( \sigma \) for a distance of 1 cm outside the coil end, denoted as “16 cm” from the midplane, one clearly recognizes the plasma wave expelled from the coil ends before the first maximum compression by the divergent lines of the increasing magnetic field. This plasma wave has already been considered in the discussion of the smear picture in Fig. 1b.

In the interior of the coil the line density remains essentially constant (the weak increase in the \( \sigma \)-values at a distance of 2.9 cm from the midplane of the coil derived from the continuum measurements is due to a slight overradiation by the probe port, which was placed nearby during these measurements). The indicated steps in the measured line densities occur with increasing delay towards the midplane of the coil. They amount to about 15—20% and therefore are within the range of error of a single line density determination, but outside the root mean square range of error of the measurements. Considered as physically real, these line density steps may well be described as a rarefaction wave moving from the ends of the coil towards its midplane. The difference in the number of particles before and after this wave has passed the observation slit will be lost for the plasma column by end losses. Actually, the velocity for this incoming wave can be deduced from Fig. 2 as having a value of about \( 1.3 \times 10^7 \) cm/s, which is in reasonable agreement with the sound velocity of about \( 1 \times 10^7 \) cm/s to be expected for it under the present conditions. If, therefore, the indicated line density steps are further interpreted as an incoming rarefaction wave the following conclusion can be drawn concerning the end losses:

End losses at time \( t \):

\[
\varphi(t) \approx \varphi = \int_{z=15.0 \text{ cm}}^{z=0.0 \text{ cm}} \frac{2\sigma(t)}{3t} \, dz \approx 3 \times 10^{23} \frac{\text{particles}}{s}
\]

The error of this value, however, must be taken fairly high, having a magnitude of about 50% because of the relatively small difference in the line density before and after the step and because of the scattering in the measured points.

Since the particles which leave the volume inside the coil as end losses must pass the plasma cross section at the end of the coil the value of the velocity \( u(z, t) \) for the outflow of the particles through the coil end (more exactly at \( z = 12.9 \) cm or at \( z = 16.0 \) cm respectively from the midplane of the coil) can also be given:

\[
u(z, t) \approx u = q/\rho \approx 4 \times 10^7 \text{ cm/s}.
\]

The uncertainty of this value is again about ±50%, mainly due to the uncertainty in \( \varphi \). This value gives the macroscopic velocity referred to all particles in the plasma cross section at the coil end and is superimposed on the thermal velocity distribution. It is about one half of the sound velocity in agreement with one-dimensional calculations for the outflow into a region of very low pressure based on a crudely approximating model.

B) \( B_{z0} = -2.3 \) kG, \( p(H_2) = 0.10 \) torr

After the results for a trapped parallel magnetic field those for an antiparallel magnetic field are given. Here the long time history behaviour of the plasma is governed by the fact that the magnetic field inside the plasma is opposite in direction compared with the driving one. As a result the magnetic field lines join each other about the coil ends, and a driving force therefore acts on the plasma in these regions in the direction towards the midplane of the coil. Hence a contraction wave collects the plasma particles at the ends of the plasma column and carries them inwards. The purpose of the investigations was to measure the collection rate of this contraction wave.

In the course of previous studies, however, it had already been shown that a plasma column with an antiparallel trapped magnetic field has a tendency to break up into several separate rings, because the

magnetic field penetrated the plasma at certain regions along the plasma column. It was demonstrated that this breakup is favoured by probe ports inserted in the plasma, by side pockets on the discharge tube extending radially outwards and by water deposits on the inner tube walls originating from the preceding discharges. For the measurements discussed here a smooth discharge tube was applied only, and the water contaminations of the inner tube walls were kept to a minimum by preceding cleaning discharges as described in 4. Fig. 3 shows a smear picture of such a discharge, in which the plasma behaviour along the tube axis was observed as explained in Fig. 1 b. This demonstrates that it was possible to avoid the breakup of the plasma column and it also proves that the occurrence of the contraction wave moving inwards was the only event of axial motion.

The number of particles collected by this contraction wave was again derived from continuum measurements at a distance of \( z = 7.9 \) cm from the midplane of the coil, that is about half way between the latter and the end of the coil. The intensity profiles were corrected for the background intensity in the way already described in the case of the parallel trapped magnetic field. The results of the measurements are shown in Fig. 4. First a smear picture is given (Fig. 4 a), which was taken in the way illustrated by Fig. 1 a. This demonstrates the radiation from the different volume elements across the plasma diameter before, during and after the contraction wave passes the observation slit. At about the moment of the first maximum compression oxygen lines — mainly emitted by O II ions — radiate strongly from volume elements behind the imploding plasma wave and apparently enlarge the diameter of the latter. In the more purified discharges, as used in the described measurements, this additional seam of impurity radiation was absent but the exposure of the smear pictures was then too low for reproductions.

In the lower part of the figure, i.e., in Fig. 4 c, the radially resolved electron densities measured are plotted for several times. These density profiles prove that the plasma column retains its hollow structure for the whole of the period until the contraction wave has passed the observation slit. The time dependence of the line density

\[
\sigma = 2\pi \int N_e r \, dr \quad \text{at } r = 0.3 \text{ cm}
\]

Fig. 4. Example of a smear photo across the plasma diameter and experimental results of the line density and electron density profiles for an antiparallel trapped magnetic field at \( z = 7.9 \) cm from the midplane of the coil.

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is shown in Fig. 4b. Again the value \( \sigma_0 \) is noted which is deduced from the used hydrogen filling pressure \( p(H_2) = .10 \) torr.

Since the velocity of the contraction wave moving towards the midplane of the coil is found from the smear picture, Fig. 3, to be \( u = 1 \times 10^7 \) cm/s the number of particles collected by it can be calculated from the increase of the line density while the contraction wave passes the observation slit. This gives

\[
\Delta N = \int_{t_1}^{t_2} (\sigma(t) - \sigma(t_0))u \, dt = 4 \times 10^{17} \text{ particles}
\]

\( \sigma(t_0) \) here signifies the line density before the contraction wave arrives at the location of the observation slit. Its value is \( \sigma(t_0) = 1.1 \times 10^{17} \) cm\(^{-1} \) according to Fig. 4b. As in the case of a parallel trapped magnetic field the line density at the beginning of the main discharge decreased towards the coil ends due to end losses during the preionization. It was found that the number of particles in the volume between \( z = 7.9 \) cm and \( z = 15.0 \) cm before the beginning of the main discharge was:

\[
N = \int_{z=7.9 \text{ cm}}^{z=15.0 \text{ cm}} \sigma(t_0, z) \, dz = 5 \times 10^{17} \text{ particles}
\]

From a comparison of the two figures given the collection rate of the contraction wave can be derived: \( \Delta N/N = .80 \pm .15 \).

In agreement with this result is the number of particles left behind the contraction wave. At \( z = 12.9 \) cm the line density after the wave had completely passed this place was found to be \( \sigma = .2 \times 10^{17} \) cm\(^{-1} \). Assuming this value to be valid everywhere behind the contraction wave, a total of \( N' \approx 1 \times 10^{17} \) particles is left in the volume between the end of the wave and that of the coil.

Recently, the axial and radial time behaviour of the plasma column in the case of an antiparallel trapped magnetic field was treated theoretically by a two-dimensional magnetohydrodynamic computer programme \(^5\). These calculations also yielded a contraction wave moving towards the midplane of the coil with a velocity which is in agreement with the experimental findings.

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