To calculate the contribution of this diagram, we have used for the (j z !N*) -vertex the isobaric model

\[ F_3(q^2) = \exp - \left( \frac{q^2}{\pi} + \frac{\delta_1(\sigma^2)}{4\pi} \int_{4\pi}^{\infty} \delta_1(\sigma^2) \, d\sigma^2 \right) \]  

(7)

where \( \delta_1(\sigma^2) \) is the d-wave \( \pi \pi \)-scattering phase shift.

At the resonance there holds

\[ \delta_1(q^2) = \pi/2 \quad (q^2 = m_0^2) \]

With the assumptions used in 13–15 we find that

\[ \Delta_{\text{theoret.}} \sim -0.079 \quad \text{for} \quad t = q^2 = -19.5 \, f^2 \]

The experiments 16, 17 give

\[ \Delta_{\text{exp.}} = -0.094 \pm 0.046 \quad \text{for} \quad q^2 = -19.5 \, f^2 \]

For the polarization (projection on a plane perpendicular to the coplanar reaction plane) we find a maximum value of \(-1.2\%\) for electron energies \( E_1 = 1.2 \, \text{GeV} \) and \( \cos \theta_{\text{CMS}} = \pi/2 \). The experimental results 18 are of the same order.

Electron Paramagnetic Resonance of Eu²⁺ in CdF₂

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The paramagnetic resonance spectrum of Eu²⁺ in CdF₂ (about 0.1 mole % Eu) shows the same characteristics as in other alkaline earth fluorides with a CaF₂-type lattice 1–4.

The angular variation shows that the Eu²⁺-ions in CdF₂ are in an electric field of cubic symmetry occupying Cd sites. The ground state of the Eu²⁺-ions is \( ^{8}S_7/2 \). For this case Baker, Bleaney and Hayes 2 give a spin Hamiltonian of the following form:

\[ H = g \beta \mathbf{H} \cdot \mathbf{S} + A \mathbf{I} \cdot \mathbf{S} \]

\[ + B_4(O_4^0 + 5 O_4^4) + B_6(O_6^0 - 21 O_6^4) \]

Taking this spin Hamiltonian, the splitting of the \( ^{8}S_7/2 \) ground state in a cubic crystal field (8-fold coordination) and with hyperfine interaction in the magnetic field \( H \) was calculated 5, 4, 2. The magnetic dipole transitions \( \Delta M = \pm 1 \) (without hyperfine interaction) are given by

\[
M = \pm 7/2 \rightarrow \pm 5/2 \quad [\text{abbreviated: } \pm 7/2] \quad : \quad h \tau = \Delta \varepsilon_{\pm 7/2}(H) = g \beta H \pm 7/2 + Q_7 \]

\[
Q_7 = \pm 20 p b_4 \pm 6 q b_6 + (-1 + 114 q - 345 q^2 + 84 q^3) \times 10 b_2^2 (g \beta H \pm 7/2) \]

\[
M = \pm 5/2 \rightarrow \pm 3/2 \quad [\pm 5/2] \quad : \quad h \tau = \Delta \varepsilon_{\pm 5/2}(H) = g \beta H \pm 5/2 + Q_5 \]

\[
Q_5 = \mp 10 p b_4 \mp 14 q b_6 + (-92 q + 455 q^2 - 441 q^3) \times 20 b_2^2 (3 g \beta H \pm 5/2) \]

(1)

\[ M = \pm 3/2 \rightarrow \pm 1/2 \quad [ \pm 3/2 ]:\]

\[ h\nu = \Delta E_{\pm 3/2}(H) = \gamma \beta H_{\pm 3/2} + Q_3, \]

\[ Q_3 = \mp 12 p b_4 \pm 14 q b_6 + (1 + 78 \varphi - 423 \varphi^2 + 588 \psi) 10 b_4^2/(\gamma \beta H_{\pm 3/2}); \]

\[ M = -1/2 \rightarrow +1/2 \quad [1/2 ]:\]

\[ h\nu = \Delta E_{\pm 1/2}(H) = \gamma \beta H_{\pm 1/2} + Q_1, \]

\[ Q_1 = (7 - 462 \varphi + 2175 \varphi^2 - 2940 \psi) 5 b_4^2/(2 \gamma \beta H_{1/2}), \]

where \[ b_4 = 60 B_4; \quad b_6 = 1260 B_6; \quad \varphi = F m^2 + m^2 n^2 + n^2 p^2 \quad (l, m, n = \text{direction cosines}); \]

\[ p = 1 - 5 \varphi; \quad 2 q = 2 - 21 \varphi + 231 \psi. \]

We must add to Eq. (1) the following terms for hyperfine interaction:

\[ A m + (A^2/4 h\nu) \{ F(M) f(-m) + F(-M + 1) f(m) - F(-M) f(m) \}
\]

\[ + F(M - 1) f(-m) h\nu[\Delta E_{M+1}(H_M)]^{-1} - F(M - 1) f(-m), \]

where \[ F(M) = (J + M)(J - M + 1) \quad \text{and} \quad f(m) = (I + m)(I - m + 1). \]

\[ F = \text{electron spin}; \quad I = \text{nuclear spin}; \quad (\text{in our case } J = S = 7/2, I = 5/2). \]

With Eqs. (1) and (2) we obtain the following line positions for the transitions \( \Delta M = \pm 1, \Delta m = 0 \)

\[ M = \pm 7/2, m: \]

\[ g \beta H_{\pm 7/2, m} = h\nu - Q_7 - A m - (A^2/4 h\nu) \quad \{14(35 - 4 m^2) - 12(35 - 4 m^2 - 4 m) h\nu[\Delta E_{\pm 3/2}(H_{\pm 7/2})]^{-1}; \]

\[ M = \pm 5/2, m: \]

\[ g \beta H_{\pm 5/2, m} = h\nu - Q_5 - A m - (A^2/4 h\nu) \quad \{24(35 - 4 m^2) - 15(35 - 4 m^2 + 4 m) h\nu[\Delta E_{\pm 3/2}(H_{\pm 5/2})]^{-1} \}
\]

\[ - 7(35 - 4 m^2 \pm 4 m) h\nu[\Delta E_{\pm 7/2}(H_{\pm 5/2})]^{-1}; \]

\[ M = \pm 3/2, m: \]

\[ g \beta H_{\pm 3/2, m} = h\nu - Q_3 - A m - (A^2/4 h\nu) \quad \{30(35 - 4 m^2) - 16(35 - 4 m^2 + 4 m) h\nu[\Delta E_{\pm 3/2}(H_{\pm 3/2})]^{-1} \}
\]

\[ - 12(35 - 4 m^2 \pm 4 m) h\nu[\Delta E_{\pm 5/2}(H_{\pm 3/2})]^{-1} \}; \]

\[ M = 1/2, m: \]

\[ g \beta H_{1/2, m} = h\nu - Q_1 - A m - (A^2/4 h\nu) \quad \{32(35 - 4 m^2) - 15(35 - 4 m^2 - 4 m) h\nu[\Delta E_{\pm 3/2}(H_{1/2})]^{-1} \}
\]

\[ - 15(35 - 4 m^2 \pm 4 m) h\nu[\Delta E_{\pm 5/2}(H_{1/2})]^{-1} \}. \]

We see that Eq. (3) in which fine structure splitting and hyperfine structure splitting are considered up to terms of 2. order in \( b_4 \) and \( A \) are sufficient to explain the spectrum, especially the angular variation. The constants of Eq. (3), i.e. \( g, A(\text{Eu}^{151}), A(\text{Eu}^{153}), b_4 \) and \( b_6 \) were calculated from the spectrum for \( H || [100] \)-direction and verified with Eq. (3) on the spectrum for \( H || [111] \)-direction.

The spectrum of Eu\(^{2+}\) in CdF\(_2\) for \( H || [100] \)-direction is shown in Fig. 1. The hyperfine lines of the transition \( M = -1/2 \rightarrow +1/2 \) are overlapped by the spectrum of manganese unintentionally present in the crystal.

In Table 1 the EPR-constants of the system CdF\(_2\): Eu are compared with published values of EPR-constants of the Eu\(^{2+}\)-ion in alkaline earth fluorides having the

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**Fig. 1.** The EPR-spectra of Eu\(^{2+}\) and Mn\(^{2+}\) in CdF\(_2\) for \( H || [100] \)-direction.
same lattice structure (space group $Oh^5$) as CdF$_2$. We are not surprised that our constant $b_4$ describing the cubic part of the lattice potential agrees with that found by Rytter$^1$ in the system CaF$_2$: Eu. Both host lattices have nearly equal lattice dimensions. Baker et al.$^2$ received a value for $b_4$ in the system CaF$_2$ somewhat higher. But we believe Rytter did his measurements contrary to Baker et al. at a temperature of 300 °K. Table 1 shows that the values $b_4$ for the systems SrF$_2$: Eu and BaF$_2$: Eu are always higher at low temperatures. This can be explained by shrinkage of the lattice$^3$.

The hyperfine constants $A$(Eu$^{151}$) and $A$(Eu$^{153}$) in CdF$_2$ are larger than in CaF$_2$. After van Wieringen$^8$ this means the covalent part of bonding to be greater in CaF$_2$ than in CdF$_2$.

The widths of the hyperfine structure lines of the isotop Eu$^{151}$ in CdF$_2$, measured between the turning points of the lines, are $(12 \pm 1)$ G for the transitions $M = \pm 7/2 \rightarrow \pm 5/2$, $(10 \pm 1)$ G for $M = \pm 5/2 \rightarrow \pm 3/2$ and $(7 \pm 1.5)$ G for $M = \pm 3/2 \rightarrow \pm 1/2$. The line width of the transition $M = -1/2 \rightarrow +1/2$ could not be measured because of the overlapping Mn-spectrum.

We also noticed that the line widths are dependent on the angle. The lines are relatively broad, but this can be explained by unresolved superhyperfine interaction of the 8 fluor ligands with the electron spin of the Eu$^{2+}$-ion. Vinokurov et al.$^4$ published a line width of 6 G for the hyperfine structure line of Eu$^{2+}$ in BaF$_2$ and SrF$_2$ at a temperature of 77 °K. This conforms with our EPR-measurements$^7$ of manganese in CdF$_2$ compared with several publications$^8,9$ about manganese in alkaline earth fluorides which show that the SHF-interaction at room temperature is larger in CdF$_2$ than in alkaline earth fluorides. The product of line width and line height yields the theoretically expected proportions of the line intensities for these transitions.

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**Table 1. EPR data for the Eu$^{2+}$-ion in some single crystals of the space group $Oh^5$.** Simultaneous change of the signs of $A_1$ and $b_6$ leaves the line positions unaltered. * This paper.

<table>
<thead>
<tr>
<th>Host lattice</th>
<th>Nearest neighbour distance [Å]</th>
<th>Frequency [GHz]</th>
<th>Temperature [°K]</th>
<th>$g$</th>
<th>$A$(Eu$^{151}$)</th>
<th>$A$(Eu$^{153}$)</th>
<th>$b_4$</th>
<th>$-b_6$</th>
<th>ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>CdF$_2$</td>
<td>2.33</td>
<td>9</td>
<td>300</td>
<td>1.988 $\pm$ 0.002</td>
<td>36.1 $\pm$ 0.47</td>
<td>15.95 $\pm$ 0.47</td>
<td>55.5 $\pm$ 0.25</td>
<td>0.28 $\pm$ 0.2</td>
<td>*</td>
</tr>
<tr>
<td>CaF$_2$</td>
<td>2.361</td>
<td>9</td>
<td>90</td>
<td>1.9927 $\pm$ 0.001</td>
<td>34.1 $\pm$ 0.1</td>
<td>15.1 $\pm$ 0.1</td>
<td>55.6</td>
<td>0.27</td>
<td>1</td>
</tr>
<tr>
<td>CaF$_2$</td>
<td>2.361</td>
<td>24</td>
<td>90</td>
<td>1.989 $\pm$ 0.002</td>
<td>34.5 $\pm$ 0.2</td>
<td>15.3 $\pm$ 0.4</td>
<td>57.9 $\pm$ 0.2</td>
<td>0.5 $\pm$ 0.2</td>
<td>2</td>
</tr>
<tr>
<td>SrF$_2$</td>
<td>2.54</td>
<td>40</td>
<td>300</td>
<td>1.9922 $\pm$ 0.0005</td>
<td>33.9 $\pm$ 0.3</td>
<td>15.1 $\pm$ 0.3</td>
<td>44.9 $\pm$ 0.5</td>
<td>0.24 $\pm$ 0.5</td>
<td>3</td>
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<tr>
<td>SrF$_2$</td>
<td>2.54</td>
<td>40</td>
<td>77</td>
<td>1.993 $\pm$ 0.001</td>
<td>33.8 $\pm$ 0.6</td>
<td>15.2 $\pm$ 0.6</td>
<td>46.8 $\pm$ 0.6</td>
<td>0.2 $\pm$ 0.6</td>
<td>4</td>
</tr>
<tr>
<td>BaF$_2$</td>
<td>2.663</td>
<td>40</td>
<td>300</td>
<td>1.9926 $\pm$ 0.0005</td>
<td>35.5 $\pm$ 0.3</td>
<td>15.0 $\pm$ 0.3</td>
<td>36.0 $\pm$ 0.7</td>
<td>0 $\pm$ 0.7</td>
<td>3</td>
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<tr>
<td>BaF$_2$</td>
<td>2.663</td>
<td>40</td>
<td>77</td>
<td>1.993 $\pm$ 0.001</td>
<td>33.5 $\pm$ 0.3</td>
<td>14.95 $\pm$ 0.6</td>
<td>36.7 $\pm$ 0.6</td>
<td>0.067 $\pm$ 0.6</td>
<td>4</td>
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7 To be published.
9 W. Low, Phys. Rev. 105, 793 [1957].