The Influence of the $f_0$ Meson in the Two-Photon Exchange on the Relative Ratio of $\sigma_{e^-p}$ to $\sigma_{e^+p}$ and the Polarization of the Recoil Protons in Elastic Electron-Proton Scattering

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In a former paper we have calculated the two-photon exchange contributions in elastic $e(p)$-scattering processes. With this result we have calculated the correction term in $\sigma_0$ to the Rosenbluth formula (interference term of the one- and two-photon exchange amplitude) and that to the proton-antiproton annihilation in an electron-positron pair. Using the procedure given by Hees and Leader, an expansion of the photon-nucleon vertex was given in terms of the Compton scattering amplitude for the proton. One can neglect the cut in the $s$-plane we have considered dispersion relations in the $t$-channel only. The corrections to the Rosenbluth formula and those to the $p\bar{p}$ annihilation into an $e^-e^+$-pair are smaller by a factor $\alpha$ than the contributions of the one-photon exchange diagram; so we got a deviation of $\sim 1_5^4$ for a point nucleon. One can have only a possible influence in the resonances of the Compton scattering amplitude. Let $A_B$ be the contribution of the one-photon exchange amplitude and $A_C$ that for the two-photon case. Then near the resonance energy $2\omega \approx 300$ MeV for Compton scattering $A_C$ is predominantly imaginary, being just the shadow of the channel for the photon meson production in the resonant 3-3 channel, as Drell, Ruder mann, and Fubini have shown. Therefore $A_C$ is approximately $\pi/2$ out of phase with $A_B$, a real potential scattering term in Born approximation, and the interference term in

$$\frac{d\sigma_{e^-p}}{d\Omega} \sim |A_B + \alpha^2 A_C|^2$$

$$\approx \alpha^2 |A_B|^2 + 2 \alpha^2 \text{Re} A_B^* A_C + \ldots$$

(1)

is actually very small.\(^7, 8\)

Flamm and Kemmer have calculated the influence of $A_C$ by introducing a tensor resonon model for Compton scattering. The result is, that the maximal deviation from the straight line behaviour as a function of $\tan^2(\theta/2)$

$$A(t) + B(t) \tan^2(\theta/2)$$

appears only at very small scattering angles $\theta$.

The deviation from the Rosenbluth formula comes out to be $\sim 10%$ for an impulse transfer of $t = -30 f^{-2}$ and small $\theta$ and the assumption, that the coupling constants used in this model are of the order of 1. We find, that the deviations from (2) give a correction of $7%$ for $t = -30 f^{-2}$ and $\theta = 5^\circ$ and the $f_0$ meson in the intermediate state, using dispersion relation methods.

But if we introduce the Pomeron exchange for the $f_0$ meson, the correction amounts to be $12%$. For the coupling constants we have assumed $f_{kn}^0 = f_{kn}^0$ and $f_{kn}^0 = f_{kn}^0$ in both cases.

In order to get quantities for a better discussion in experiments and to have a direct measure for the contribution of $A_C$, we have calculated from (1) the relative ratio

$$A = \frac{\sigma_{e^-p} - \sigma_{e^+p}}{\sigma_{e^-p} + \sigma_{e^+p}} = 2 \alpha \frac{\text{Re} A_B^* A_C}{|A_B|^2}$$

(3)

and the polarization $P$ of the recoil protons in the elastic (e,p)-scattering processes, using unpolarized electrons:\(^8, 13\)

$$P = \frac{\text{Tr.} \{ M^* \sigma \cdot s M \}}{\text{Tr.} \{ M^* M \}}.$$  

(4)

$M$ denotes the transition matrix, $\sigma$ the Pauli matrices and $s$ the spin direction of the recoil proton ($s^2 = 1$). Expanding the $S$-matrix in powers of $\alpha$, we get

$$S = 1 + i M_2 + i M_4 + \ldots ,$$

(5)

where $M_2$ corresponds to the one-photon exchange term $A_B$ (hermitian!) and $M_4$ to the $A_C$. To lowest order in $\alpha$, the polarization is then given by

$$P = 2 i \frac{\text{Tr.} \{ M^* (\sigma \cdot s) \text{ Im} M_4 \}}{\text{Tr.} \{ M_2^* M_4 \}} .$$

(6)

In order to calculate the $M_4$ resp. $A_C$, the unitarity condition, calculated for the channels given in Fig. 1, was used.


\(^2\) A. C. Hees and E. Leader, Phys. Rev. 126, 789 [1962].

\(^3\) S. D. Drell and S. Fubini, Phys. Rev. 113, 741 [1959].


\(^6\) S. D. Drell and M. A. Ruder mann, Phys. Rev. 106, 561 [1957].

\(^7\) S. D. Drell, Form Factors of Elementary Particles, Proc. Intern. School of Physics, Enrico Fermi, Course XXVI, p. 206/207.


\(^10\) M. Goubin, Nuovo Cim. 21, 1094 [1961].


\(^12\) W. Kemmer, CERN, 3272/T. H. 255 (13. 3. 1962).

To calculate the contribution of this diagram, we have used for the \((\pi N N^*)\)-vertex the isobaric model of \textsc{Gourdin} and \textsc{Salin}\textsuperscript{14, 15}, who calculated the photo production processes.

Further, to calculate \(\text{Im} M_4\) for the channels given in Fig. 1, a technique similar to that given in \textsuperscript{13} was used. To obtain the form factor in the \((\gamma \pi)\)-vertex, the \(f_0\) resonance was introduced and for the pion form factor a subtracted dispersion relation \textsuperscript{7} was used, viz.:

\[
F_\pi(q'^2) = \exp \left[ \int \frac{q'^2}{\pi} \frac{4\pi}{q'^2} \frac{\delta_1(\sigma^2) d\sigma^2}{(\sigma^2-q'^2-\epsilon - i \epsilon)} \right],
\]

where \(\delta_1(\sigma^2)\) is the d-wave \(\pi\pi\)-scattering phase shift.

At the resonance there holds

\[
\delta_1(q^2) = \pi/2 \quad (q^2 = m^2).
\]

With the assumptions used in \textsuperscript{13-15} we find that \(\Delta_{\text{theoret}} \approx -0.079\) for \(t = q^2 = -19.5 f^{-2}\).

The experiments \textsuperscript{16, 17} give

\[
\Delta_{\text{exp}} = -0.094 \pm 0.046 \quad \text{for} \quad q^2 = -19.5 f^{-2}.
\]

For the polarization (projection on a plane perpendicular to the coplanar reaction plane) we find a maximum value of \(-1.2\%\) for electron energies \(E_e = 1.2\text{GeV}\) and \(\cos \theta_{\text{cm}} = 1/2.\) The experimental results \textsuperscript{18} are of the same order.

\textsuperscript{14} \textsc{M. Gourdin} and \textsc{Ph. Salin}, Nuovo Cim. \textbf{27}, 1, 193 [1963].
\textsuperscript{15} \textsc{M. Gourdin} and \textsc{Ph. Salin}, Nuovo Cim. \textbf{27}, 309 [1963].
\textsuperscript{17} \textsc{A. Browman}, \textsc{F. Liu}, and \textsc{C. Schaefer}, Phys. Rev. Letters \textbf{7}, 183 [1964].
\textsuperscript{18} \textsc{J. C. Bizot}, \textsc{J. Buoz}, \textsc{J. LeFrancois}, \textsc{J. Perez-Y-Jorba}, and \textsc{P. Roy}, Sienna Intern. Conf. Elementary Particles (1963), Abstract No. 139 and Phys. Rev. Letters \textbf{11}, 10, 480 [1963].

\section*{Electron Paramagnetic Resonance of Eu\textsuperscript{2+} in CdF\textsubscript{2}}

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The paramagnetic resonance spectrum of Eu\textsuperscript{2+} in CdF\textsubscript{2} (about 0.1 mole \% Eu) shows the same characteristics as in other alkaline earth fluorides with a CaF\textsubscript{2}-type lattice\textsuperscript{1-4}.

The angular variation shows that the Eu\textsuperscript{2+}-ions in CdF\textsubscript{2} are in an electric field of cubic symmetry occupying Cd sites. The ground state of the Eu\textsuperscript{2+}-ions is \(8S_{7/2}\). For this case \textsc{Baker}, \textsc{Bleaney} and \textsc{Hayes}\textsuperscript{2} give a spin \textsc{H}amiltonian of the following form:

\[
H = g \beta H \cdot S + A I \cdot S + B_4 (O_4 + 5 O_4^2) + B_6 (O_6 - 21 O_6^2).
\]

Taking this spin \textsc{H}amiltonian, the splitting of the \(6S_{7/2}\) ground state in a cubic crystal field (8-fold coordination) and with hyperfine interaction in the magnetic field \(H\) was calculated \textsuperscript{2, 4, 5}. The magnetic dipole transitions \(\Delta M = \pm 1\) (without hyperfine interaction) are given by

\[
M = \pm 7/2 \rightarrow \pm 5/2 \quad \text{[abbreviated: \(\pm 7/2\)]:} \quad h v = \Delta E_{\pm 7/2}(H) = g \beta H \pm 7/2 + Q_7,
\]
\[
M = \pm 5/2 \rightarrow \pm 3/2 \quad \text{[\(\pm 5/2\)]:} \quad h v = \Delta E_{\pm 5/2}(H) = g \beta H \pm 5/2 + Q_5,
\]

where

\[
Q_7 = 100 p b_4 + 14 q b_6 + (92 \varphi + 455 q^2 - 441 \psi) 20 b_4^2 \quad (3 g \beta H \pm 5/2).
\]

\textsuperscript{1} \textsc{C. Ryter}, Helv. Phys. Acta \textbf{30}, 353 [1957].
\textsuperscript{2} \textsc{J. M. Baker}, \textsc{B. Bleaney}, and \textsc{W. Hayes}, Proc. Roy. Soc., Lond. A \textbf{247}, 141 [1958].
\textsuperscript{3} \textsc{R. S. Titte}, Phys. Letters \textbf{6}, 13 [1963].

\textsuperscript{4} \textsc{V. M. Vinokurov}, \textsc{M. M. Zaripov}, \textsc{V. G. Stepakov}, \textsc{G. K. Churkin}, and \textsc{L. Ya. Shekun}, Soviet Phys. —Solid State \textbf{5}, 1415 [1964].

\textsuperscript{5} \textsc{R. Lacroix}, Helv. Phys. Acta \textbf{30}, 374 [1957].