The Influence of the \( f_0 \) Meson in the Two-Photon Exchange on the Relative Ratio of \( e^-p \) to \( e^+p \) and the Polarization of the Recoil Protons in Elastic Electron-Proton Scattering

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In a former paper 1 we have calculated the two-photon exchange contributions in elastic (\( e,p \))-scattering processes. With this result we have calculated the correction term in \( z^2 \) to the Rosenbluth formula (interference term of the one- and two-photon exchange amplitude) and that to the proton-antiproton annihilation in an electron-positron pair. Using the procedure given by Heann and Leader 2, an expansion of the photon-nucleon vertex was given in terms of the Compton scattering amplitude for the proton. Because one can neglect the cut in the s-plane 3 we have considered dispersion relations in the \( t \)-channel only. The corrections to the Rosenbluth formula and those to the \( p \bar{p} \) annihilation into an \( e^-e^+ \)-pair are smaller by a factor \( z \) than the contributions of the one-photon exchange diagram; so we got a deviation of \( \sim 1.5 \% \) for a point nucleon. One can only have a possible influence in the resonances of the Compton scattering amplitude. Let \( A_0 \) be the contribution of the one-photon exchange amplitude and \( A_C \) that for the two-photon case. Then near the resonance energy \( \omega_R \approx 300 \text{ MeV} \) for Compton scattering \( A_C \) is predominantly imaginary, being just the shadow of the channel for the photon meson production in the resonant 3-3 channel, as Dreil, Rudermann 4 and Furry 5 have shown. Therefore \( A_C \) is approximately \( \pi/2 \) out of phase with \( A_0 \), a real potential scattering term in Born approximation, and the interference term in

\[
\frac{d\sigma}{d\Omega} \sim |A_0 + z^2 A_C|^2
\]

is actually very small 7,8.

Flamm and Kümmer 9 have calculated the influence of \( A_C \) by introducing a tensor resonance model for Compton scattering. The result is, that the maximal deviation from the straight line behaviour as a function of \( \tan^2(\theta/2) \) is 11

\[
A(t) + B(t) \tan^2(\theta/2) \]

appears only at very small scattering angles \( \theta \).

The deviation from the Rosenbluth formula comes out to be \( \sim 10 \% \) for an impulse transfer of \( t = -30 \text{ f}^{-2} \) and small \( \theta \) and the assumption, that the coupling constants used in this model are of the order of 1. We find 3, that the deviations from (2) give a correction of \( 7 \% \) for \( t = -30 \text{ f}^{-2} \) and \( \theta = 5 \degree \) and the \( f_0 \) meson in the intermediate state, using dispersion relation methods.

But if we introduce the Pomeranchuck trajectory 12 for the \( f_0 \) meson, the correction amounts to be 12%. For the coupling constants we have assumed \( f_{e^+e^-} = f_{e^-e^+} \) and \( f_{e^+e^-} = f_{e^-e^+} \) in both cases.

In order to get quantities for a better discussion in experiments and to have a direct measure for the contribution of \( A_C \), we have calculated from (1) the relative ratio

\[
A = \frac{\sigma_{e^-p} - \sigma_{e^+p}}{\sigma_{e^-p} + \sigma_{e^+p}} = 2 \frac{\text{Re} A^*_{\bar{p}} A_C}{|A_0|^2} \]

and the polarization \( P \) of the recoil protons in the elastic (\( e,p \))-scattering processes, using unpolarized electrons 8,13

\[
P = \frac{\text{Tr.} (M^* \sigma M)}{\text{Tr.} (M^* M)}. \]

\( M \) denotes the transition matrix, \( \sigma \) the Pauli matrices and \( \sigma \) the spin direction of the recoil proton (\( \sigma_z = 1 \)).

Expanding the S-matrix in powers of \( z \), we get

\[
S = 1 + i M_4 + i M_4 + \ldots, \]

where \( M_4 \) corresponds to the one-photon exchange term \( A_B \) (hermitian!) and \( M_4 \) to the \( A_C \). To lowest order in \( z \), the polarization is then given by

\[
P = 2 \left( 1 + \frac{\text{Tr.} (M_4 (\sigma \bar{\sigma} \sigma M_4)}{\text{Tr.} (M^* M_4)} \right). \]

In order to calculate the \( M_4 \) resp. \( A_C \), the unitarity condition, calculated for the channels given in Fig. 1, was used.

3 S. D. Dreil and S. Furry, Phys. Rev. 113, 741 [1959].
7 S. D. Dreil, Form Factors of Elementary Particles, Proc. Intern. School of Physics, Enrico Fermi, Course XXVI, p. 206/207.
To calculate the contribution of this diagram, we have used for the \((\pi N N^*)\)-vertex the isobaric model of Gourdin and Salin, who calculated the photo production processes.

Further, to calculate \(\text{Im}\, M_4\) for the channels given in Fig. 1, a technique similar to that given in 13 was used. To obtain the form factor in the \((\gamma N\pi)\)-vertex, the \(f_0\) resonance was introduced and for the pion form factor a subtracted dispersion relation 7 was used, viz.:

\[
F_\pi(q^2) = \exp \left[ \frac{q^2}{\pi} \int_{4m^2}^{\infty} \frac{\delta_1(\sigma^2)}{\sigma^2(q^2-\sigma^2-i\epsilon)} \right],
\]

where \(\delta_1(\sigma^2)\) is the d-wave \(\pi\)-scattering phase shift.

At the resonance there holds

\[
\delta_1(q^2) = \pi/2 \quad (q^2 = m_{f_0}^2).
\]

With the assumptions used in 13–15 we find that

\[
\mathcal{A}_{\text{theoret.}} = -0.079 \quad \text{for} \quad q^2 = -19.5\,f^{-2}.
\]

The experiments 16, 17 give

\[
\mathcal{A}_{\exp.} = -0.094 \pm 0.046 \quad \text{for} \quad q^2 = -19.5\,f^{-2}.
\]

For the polarization (projection on a plane perpendicular to the coplanar reaction plane) we find a maximum value of \(-1.2\%\) for electron energies \(E_e = 1.2\,\text{GeV}\) and \(\cos \theta = \frac{t}{g^2}\). The experimental results 18 are of the same order.

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**Electron Paramagnetic Resonance of \(\text{Eu}^{2+}\) in \(\text{CdF}_2\)**

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The paramagnetic resonance spectrum of \(\text{Eu}^{2+}\) in \(\text{CdF}_2\) (about 0.1 mole % Eu) shows the same characteristics as in other alkaline earth fluorides with a \(\text{CaF}_2\)-type lattice.

The angular variation shows that the \(\text{Eu}^{2+}\)-ions in \(\text{CdF}_2\) are in an electric field of cubic symmetry occupying \(\text{Cd}\) sites. The ground state of the \(\text{Eu}^{2+}\)-ions is \(8S_{7/2}\). For this case Baker, Bleaney, and Hayes 2 give a spin Hamiltonian of the following form:

\[
H = g\beta H \cdot S + A I \cdot S + B_4(O_4^2 + 5O_4 + O_6^2 - 21O_6^2).
\]

Taking this spin Hamiltonian, the splitting of the \(6S_{7/2}\) ground state in a cubic crystal field (8-fold coordination) and with hyperfine interaction in the magnetic field \(H\) was calculated 2, 4, 5. The magnetic dipole transitions \(\Delta M = \pm 1\) (without hyperfine interaction) are given by

\[
\begin{align*}
M = \pm 7/2 &\rightarrow \pm 5/2 \quad \text{[abbreviated: } \pm 7/2]\quad : \quad h\nu = \Delta E = \pm 7/2(H) = g\beta H \pm 7/2 + Q_1, \\
Q_1 &= \pm 20 p b_4 \pm 6 q b_6 + (-1 + 114 \varphi - 345 \varphi^2 + 84 \psi) 10 b_4^2/(g\beta H \pm 7/2); \\
M = \pm 5/2 &\rightarrow \pm 3/2 \quad \text{[}\pm 5/2]\quad : \quad h\nu = \Delta E = \pm 5/2(H) = g\beta H \pm 5/2 + Q_5, \\
Q_5 &= \pm 10 p b_4 \pm 14 q b_6 + (-92 \varphi + 455 \varphi^2 - 441 \psi) 20 b_4^2/(3g\beta H \pm 5/2).
\end{align*}
\]

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