As the absolute intensities of Raman lines due to the symmetrical $\text{C}=\text{N}$, $\text{C}-\text{C}$ and $\text{C}=\text{C}$ stretching vibrations depend on the derivatives of the polarizabilities of the respective bonds, such calculations have been made here from the above derived equation using the internuclear distances, delta-function strengths, Born radius, etc. given earlier for carbon subnitride.

The calculated values of the polarizability derivatives in $\text{Å}^2$ for the $\text{C}=\text{N}$, $\text{C}-\text{C}$ and $\text{C}=\text{C}$ bonds are 2.347, 1.227 and 2.411, respectively. These values compare well with those measured experimentally in other related systems having similar chemical bonds. They are 2.61 $\text{Å}^2$ for the $\text{C}=\text{N}$ bond in acetonitrile.

In the past few years several papers have been published dealing with the problem of the scattering of electromagnetic waves in a plasma. The radiation energy $dI_2(\omega_2, \mathbf{k}_2)$ with the frequency $\omega_2$ and the wavevector $\mathbf{k}_2$ that is scattered per second into a given solid angle $d\Omega$ is given by

$$dI_2(\omega_2, \mathbf{k}_2) = \lim_{T \to \infty} \frac{1}{T} I_1(\omega_1, \mathbf{k}_1) \left( \frac{n(\mathbf{k}, \omega)}{2} \right) d\omega_2 d\mathbf{k}_2$$

where $I_1(\omega_1, \mathbf{k}_1)$ is the primary intensity with the frequency $\omega_1$, and the wave vector $\mathbf{k}_1$, $\mathbf{n}$ is the scattering crosssection for a single electron and $T$ is the duration of observation. The ensemble average is denoted by $\langle \cdot \rangle$ and $n(\mathbf{k}, \omega)$ is the Fourier transform of the electron density, where $\mathbf{k}$ and $\omega$ are given by

$$\mathbf{k}_2 = \mathbf{k}_1 + \mathbf{k}$$
$$\omega_2 = \omega_1 + \omega$$

(2a)

or

$$\mathbf{k}_2 = \mathbf{k}_1 - \mathbf{k}$$
$$\omega_2 = \omega_1 - \omega$$

(2b)

These conditions mean that in general the $\mathbf{k}$-vectors must form a triangle.

From eq. (1) and (2) follows that the spectral distribution of the scattered radiation represents the spectral distribution of the electron density fluctuations, if the primary radiation is monochromatic. By this the problem of calculating the spectrum of the scattered radiation is reduced to the problem of calculating the density fluctuations. This problem has


\[ n(k, s) = \frac{[Y_e(k, s) - S_e(k, s)] [1 - R_1(k, s)] - R_e(k, s) [Z Y_i(k, s) - S_i(k, s)]}{1 - R_e(k, s) - R_i(k, s)} , \]  

\[ N(k, s) = \frac{[Y_i(k, s) - Z^{-1} S_i(k, s)] [1 - R_1(k, s)] - Z^{-1} R_i(k, s) [Y_e(k, s) - S_e(k, s)]}{1 - R_e(k, s) - R_i(k, s)} \]

where the functions \( R, S, Y \) are defined by

\[ R_e(k, s) = - \int_{-\infty}^{\infty} d^3v \int_{-\infty}^{\infty} \frac{i A e c k \, \overline{\mathcal{F}_e(v')}}{k^2 B} \, A_e \, dq', \]  

\[ R_i(k, s) = - \int_{-\infty}^{\infty} d^3v \int_{-\infty}^{\infty} \frac{i A e c k \, \overline{\mathcal{F}_i(v')}}{k^2 B} \, A_i \, dq', \]  

\[ S_e(k, s) = - \int_{-\infty}^{\infty} d^3v \int_{-\infty}^{\infty} c \, \overline{\mathcal{F}_e(v')} \, E_{ex}(k, s) \, A_e \, dq', \]  

\[ S_i(k, s) = + \int_{-\infty}^{\infty} d^3v \int_{-\infty}^{\infty} \frac{1}{\Omega_e} \, F_1(v', k) \, A_e \, dq', \]  

\[ Y_e(k, s) = + \int_{-\infty}^{\infty} d^3v \int_{-\infty}^{\infty} \frac{1}{\Omega_i} \, F_1(v', k) \, A_i \, dq', \]  

\[ Y_i(k, s) = + \int_{-\infty}^{\infty} d^3v \int_{-\infty}^{\infty} \frac{1}{\Omega_i} \, F_1(v', k) \, A_i \, dq', \]

with

\[ A_e = \exp \left\{ - \frac{1}{\Omega_e} \left[ (s - i k u \cos \psi) (\varphi - \varphi') - i k w \sin \psi (\sin \varphi - \sin \varphi') \right] \right\} , \]  

\[ A_i = \exp \left\{ \frac{1}{\Omega_i} \left[ (s - i k u \cos \psi) (\varphi - \varphi') - i k w \sin \psi (\sin \varphi - \sin \varphi') \right] \right\} . \]

\( \Omega_e \) and \( \Omega_i \) are the electron and the ion gyrofrequency, \( \psi \) is the angle between \( k \) and \( B \) and \( v' \) is obtained from eq. (14) by replacing \( \varphi \) by \( \varphi' \). The functions \( R, S \) and \( Y \) are discussed in the appendix.

From the scattering formula (1) we see that the spectrum of the scattered light is essentially determined by the quantity

\[ Q(k, \omega) = \lim_{T \to \infty} \frac{1}{2 \pi T} \langle |n(k, \omega)|^2 \rangle \]

where the average is taken over the initial values. In order to connect the quantity \( n(k, s) \) of eq. (15) with \( Q(k, \omega) \), as defined in (21), we put

\[ s = \gamma + i \omega \]  

with \( \gamma = 1/2 T \).

By introducing the damping \( \gamma \) we take into account that the LAPEL integral (11) is extended over an infinite time interval, while the duration of observation \( T \) — and by that the total measured energy of the scattered radiation — is finite. The damping constant \( \gamma \) is chosen such that for a stationary plasma

\[ \int_0^T |n(k, t)|^2 \, dt \approx \int_0^\infty |n(k, t)|^2 \exp (-2 \gamma t) \, dt . \]

From eq. (21) — (23) follows

\[ Q(k, \omega) = \lim_{\gamma \to 0} \frac{\gamma}{\pi} \langle |n(k, s)|^2 \rangle . \]

This quantity can be calculated from eq. (15) — (20). From eq. (15) follows:

\[ Q(k, \omega) = \lim_{\gamma \to 0} \frac{\gamma}{\pi} \langle |Y_e(k, s) - S_e(k, s)|^2 \rangle \frac{1 - R_1(k, s)}{|1 - R_e(k, s) - R_i(k, s)|^2} \]

\[ + \langle |Z Y_i(k, s) - S_i(k, s)|^2 \rangle \frac{1 - R_i(k, s)}{|1 - R_e(k, s) - R_i(k, s)|^2} . \]
In performing the ensemble average in eq. (26) one averages in $Y$ over the initial values of the fluctuations and in $S$ over the phase of the external electric field $E_{ex}(\mathbf{k}, t)$ at the time $t = 0$. Because these two quantities are obviously not correlated with each other the following relations are valid:

$$
\langle |Y_i(\mathbf{k}, s) - S_i(\mathbf{k}, s)|^2 \rangle = \langle |Y_i(\mathbf{k}, s)|^2 \rangle + \langle |S_i(\mathbf{k}, s)|^2 \rangle,
$$

(27)

$$
\langle |Y_i(\mathbf{k}, s) - S_i(\mathbf{k}, s)|^2 \rangle = \langle |Y_i(\mathbf{k}, s)|^2 \rangle + \langle |S_i(\mathbf{k}, s)|^2 \rangle.
$$

(28)

From these relations follows that we can split $Q(\mathbf{k}, \omega)$ into two parts:

$$
Q(\mathbf{k}, \omega) = Q_{th}(\mathbf{k}, \omega) + Q_f(\mathbf{k}, \omega)
$$

(29)

where $Q_{th}$ corresponds to the thermal density fluctuations as treated by HAGFORS, while $Q_f$ represents the forced density fluctuations. We have that

$$
Q_f(\mathbf{k}, \omega) = \lim_{\gamma \to 0} \langle |S_e(\mathbf{k}, \omega)|^2 \rangle |1 - R_e(\mathbf{k}, \omega) + \langle |S_i(\mathbf{k}, \omega)|^2 \rangle | \frac{R_e(\mathbf{k}, \omega)}{1 - R_e(\mathbf{k}, \omega) - R_i(\mathbf{k}, \omega)} |^2
$$

(30)

The angles $\psi$, $Z_1$, and $Z_2$ denote the directions of $\mathbf{k}$ and $E_{ex}$ relative to $B$ (see appendix). Because of the assumption that $E_{ex}$ is a transverse wave ($E_{ex} \perp \mathbf{k}$), the three angles must fulfill the condition (A22). This implies that only two of these angles can be chosen independently.

II. Light Mixing and the Generation of the Second Harmonic

In the preceding section it was shown that density fluctuations may be forced by an electromagnetic wave of a given frequency $\omega_f$ and wave vector $\mathbf{k}_f$. The light of a second beam may now be scattered by these fluctuations according to eq. (1). According to eq. (2) the scattered light has the sum and the difference frequency of the two beams — i.e. light mixing occurs. If we neglect the influence of the dispersion, i.e. if we put the index of refraction to unity ($\omega_0 \leq \omega_1, \omega_2, \omega_f$), the following relation holds:

$$
\frac{\omega_1}{k_1} = \frac{\omega_2}{k_2} = \frac{\omega_f}{k_f} = c.
$$

(35)

From the conditions (2) and (35) it follows that the light mixing effect occurs only if the two light beams are parallel. It should be remarked that the scattering formula (1) is valid for both coherent and incoherent density fluctuations. This implies that in the case $\omega_1 = \omega_f$ eq. (1) describes the generation of the second harmonic ($\omega_2 = 2 \omega_1$).

4 This is a correction to the earlier statement that the difference frequency were generated in the case of anti-parallel beams.

5 Cf. e.g. W. H. KEGEL, Report IPP 6/21 [1964].
If we assume that the electromagnetic wave forcing the density fluctuations is plane and monochromatic, then in the limit \( V \to \infty \) the Fourier transform of the electron density is a \( \delta \)-function in \( \omega - \mathbf{k} \)-space. This indicates that light will be scattered in one direction only. Because of this \( \delta \)-function it seems more reasonable to ask for the total scattered energy than for the differential. From eq. (1) with (21) follows

\[
I_2 = I_1 \int_{-\infty}^{\infty} d\omega \oint dQ(\mathbf{k}, \omega) \sigma_e \quad (36)
\]

where \( I_1 \) is assumed to be perfectly monochromatic.

In order to evaluate the expression (36) we assume the plasma volume to have the form of a rectangular solid with dimensions \( a, b \) and \( L \) along the axes of the coordinate system, which is assumed to be oriented such that the wave vector \( \mathbf{k}_f \) of the electric field, forcing the density fluctuations, possesses only a \( z \)-component:

\[
E_{\text{ex}}(r, t) = E_{\text{ex}}^0 \exp \left\{ -i(\mathbf{k}_f \cdot \mathbf{r} - \omega_f t - \eta) \right\}
\]

\[
= E_{\text{ex}}^0 \exp \left\{ -i(k_z z - \omega_f t - \eta) \right\}.
\]

\( \eta \) being the phase of \( E_{\text{ex}} \) at \( t = 0, \mathbf{r} = 0 \).

With these assumptions the Fourier transform of the forcing field becomes:

\[
E_{\text{ex}}(\mathbf{k}, s) = E_{\text{ex}}^0 \exp \left\{ i(\eta) \frac{\sin(k_x \frac{1}{2} a)}{k_x} \frac{\sin(k_y \frac{1}{2} b)}{k_y} \right\} \sin^2\left(\frac{(k_z - k_t) \frac{1}{2} L}{k_z^2 k_y^2 (k_z - k_t)^2}\right).
\]

Placing this expression in eq. (34) we obtain:

\[
Q(\mathbf{k}, \omega) = \left( \lim_{\gamma \to 0} \frac{1}{i \omega t - s} \right) \hat{n}^2 E_{\text{ex}}^0 B^2 \left( e \frac{1}{m c} \right)^4 \sin^2 \psi \sin^2 \chi \sin^2 \chi_2 \sin^2\left(\frac{(k_z - k_t) \frac{1}{2} L}{k_z^2 k_y^2 (k_z - k_t)^2}\right).
\]

The \( \omega \)-integration of eq. (36) and the limiting process of eq. (39) yields the factor 1 from the first bracket in (39). Let

\[
A = \hat{n}^2 E_{\text{ex}}^0 B^2 \left( e \frac{1}{m c} \right)^4 \sigma_e \sin^2 \psi \sin^2 \chi \sin^2 \chi_2.
\]

There follows from eq. (36):

\[
I_2 = I_1 A \oint d\psi \sin\left(\frac{k_x}{2} a\right) \sin\left(\frac{k_y}{2} b\right) \sin^2\left(\frac{(k_z - k_t) \frac{1}{2} L}{k_z^2 k_y^2 (k_z - k_t)^2}\right).
\]

To simplify the integrand of (41) we make the approximation

\[
\frac{\sin^2\left(\frac{k_x}{2} a\right)}{k_x^2} = \frac{a^2}{4} \exp \left\{ -\frac{a^2}{4 \pi} k_x^2 \right\}.
\]

This approximation is such, that the integral over the left side of eq. (42) gives the same value as the integral over the function on the right side. For \( k_x = 0 \) both functions have the same value. Similar approximations apply for the other factors in eq. (41). These approximations effectively eliminate all details in the diffraction pattern arising from the assumed finite dimensions of the plasma.

We now express \( \mathbf{k} \) by means of \( k_x \) and \( k_t \) according eq. (2). If we denote by \( \mathbf{e} \) the unit vector in the direction of \( \mathbf{k}_x \), we have

\[
\mathbf{k}_x = k_x \mathbf{e} = k_t \pm \mathbf{k}.
\]

Introducing polar coordinates so that \( \mathbf{e} \) has the components

\[
e_x = \sin \theta \cos \phi; \quad e_y = \sin \theta \sin \phi; \quad e_z = \cos \theta
\]

we obtain from eqs. (41 - 44) and the assumption \( \mathbf{k}_1 \) parallel to \( \mathbf{k}_f \) the relation:

\[
I_2 = I_1 A \oint d\varphi \int_0^{2\pi} d\theta \sin \theta \frac{a^2 b^5 L^5}{64} \exp \left\{ -\frac{1}{4 \pi} \left( a^2 k_x^2 \sin^2 \theta \cos^2 \varphi \right) + b^2 k_x^2 \sin^2 \theta \sin^2 \varphi + L^2[k_x \cos \theta - (k_t \pm k_1)^2] \right\}.
\]
We now limit our treatment to the case of the second harmonic generation $k_2 = 2 k_1$. We assume also that $a \, k_1 \gg 1$. Then the integral in eq. (45) can easily be evaluated in the special case that the considered plasma volume $V$ is a cube ($a = b = L$). For this case eq. (45) yields (with $\omega_f / k_1 = c$):

$$I_2 = J_1 A \frac{\pi^2}{64} \frac{V^2}{a^2 k_1} \exp \left( \frac{k_1^2}{\pi} \frac{a^4}{L^2 - a^2} \right)$$

(46)

$$\sin^2 \psi \sin^2 \chi_1 \sin^2 \chi_2 \cdot 8.82 \cdot 10^{24}$$

where all quantities are to be taken in Gaussian units.

In the more general case $a = b = L$ but retaining the remaining previous assumptions we obtain from (45):

$$I_2 = J_1 A \frac{V^2}{64} k_1 L^2 - a^2 \exp \left[ \frac{k_1^2}{\pi} \frac{a^4}{L^2 - a^2} \right]$$

(47)

$$\Phi - \left( \frac{k_1}{\pi} \frac{a^2}{V^2 L^2 - a^2} \right)$$

where $\Phi$ denotes the error integral. An asymptotic expansion for large arguments yields again the result (46), where $a^2$ now is the cross-section of the plasma volume through which the light beam passes.

III. The Influence of the Dispersion

The situation becomes more complicated upon consideration of the influence of the dispersion. The dispersion relation

$$c^2 k^2 = \omega^2 - \omega_p^2$$

(valid if $\omega$ is large compared to the electron gyro-frequency) implies that the condition (2) cannot be fulfilled for the case of light mixing. This situation is similar to one arising in the case of harmonic generation in crystals, where the intensity of the generated radiation is a periodic function of the crystal thickness, because interference causes an essential part of the electric field to cancel. Noting that according to (48) the generation of the waves in the case of light mixing is not in phase with the propagation, we conclude that the intensity calculated from (1), (36) or (46) is (under the assumption of parallel beams) to be multiplied by the correction factor

$$\sin^2 (Ak \pm L)$$

(49)

$$\frac{(Ak \pm L)^2}{L}$$

$L$ being the path length of the light beams through the plasma and

$$\Delta k = k_1 \pm k_f - \frac{\omega_f \pm \omega_l}{c_p (\omega_f \pm \omega_l)},$$

(50)

where $c_p (\omega)$ is the phase velocity in the plasma of a light wave with frequency $\omega$. From the eqs. (46), (48), (49) and (50) we have that $I_2$ is a periodic function of $L$. Should all considered frequencies be large compared to the plasma frequency $\omega_p$, then the maximum of this function is independent of the average electron density $\bar{n}$.

In the special case of the generation of the second harmonic it follows from (48) and (50) that

$$\Delta k = - \frac{3}{4} \frac{\omega_p}{c} \frac{\omega_f}{\omega_l}.$$  

(51)

Note that for the light of a ruby laser ($\omega_f = 2.7 \cdot 10^{15}$ sec$^{-1}$) and an average electron density $\bar{n} = 7 \cdot 10^{16}$ cm$^3$ one calculates from (52) $\Delta k/2 \approx 1$ cm$^{-1}$.

In the case that $\Delta k$ or $L$ goes to zero the factor (49) becomes unity. This shows that the influence of the dispersion may be neglected in the case of small dimensions.

IV. Discussion

In this paper the problem of light mixing in a plasma is treated in a semi-linear fashion since the density fluctuations are determined from the linearized Vlasov equation, and nonlinear coupling arises only through eq. (1). This approach is a straightforward extension of the theory of light scattering from thermal fluctuations in a magnetic field. But because of this linearization our theory does not describe effects such as the resonance, occurring when the difference frequency of the two interacting light beams is the plasma frequency. There is also no interaction between the thermal and the forced density fluctuations in the framework of the presented theory. When nonlinear equations are used throughout, this interaction leads to incoherent nonlinear scattering.


$^8$ A. Salat, private communication.
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**Appendix**

The functions $R$ and $Y$ defined by (17) and (19) were treated by Hagfors and Bernstein. Under the assumption that the undisturbed distribution functions $f_0(v)$ and $F_0(v)$ are Maxwell distributions it has been shown, that the functions $R$ can be written as

$$R_e(k, s) = -\frac{4\pi n e^2}{T_e k^2} \left[ 1 - \frac{s}{\Omega_e} g_e(k, s) \right], \quad (A1)$$

$$R_i(k, s) = -\frac{4\pi n Z e^2}{T_i k^2} \left[ 1 - \frac{s}{\Omega_i} g_i(k, s) \right], \quad (A2)$$

where $T_e$ and $T_i$ are the electron and ion temperature, $\Omega_e$ and $\Omega_i$ the electron and ion gyrofrequency and $g_e$ and $g_i$ are Gordjev integrals defined by

$$g_e(k, s) = \int_0^\infty dx \exp \left[ -\frac{s x}{\Omega_e} \right] \left[ \sin^2 \psi (1 - \cos x) + \frac{1}{2} x^2 \cos^2 \psi \right] \frac{x T_e k^2}{m \Omega_e^2}, \quad (A3)$$

$$g_i(k, s) = \int_0^\infty dx \exp \left[ -\frac{s x}{\Omega_i} \right] \left[ \sin^2 \psi (1 - \cos x) + \frac{1}{2} x^2 \cos^2 x \right] \frac{x T_i k^2}{m \Omega_i^2}, \quad (A4)$$

$\psi$ being the angle between $k$ and $B$.

In order to determine the quantity $\gamma \langle |Y_e(k, s)|^2 \rangle$ we follow the method used by Salpeter by introducing at this point the effect of particle discreteness by writing

$$f^h(r, v) = \sum_{j=1}^n \delta[r - r_j(t=0)] \cdot \delta[v - v_j(t=0)]. \quad (A5)$$

Using this in eq. (19) one deduces after some calculation:

$$\gamma \langle |Y_e(k, s)|^2 \rangle = \frac{n_v}{\Omega_v} \Re \{ g_e(k, s) \}. \quad (A6)$$

In the limit $\gamma \to 0$ this becomes

$$\lim_{\gamma \to 0} \gamma \langle |Y_e(k, s)|^2 \rangle = \frac{V \times T_e k^2}{4 \pi \omega e^2} \Re \langle R_e(k, \omega) \rangle. \quad (A7)$$

The corresponding relations for the ions are

$$\gamma \langle |Y_i(k, s)|^2 \rangle = \frac{n_i}{\Omega_i} \Re \{ g_i(k, s) \}, \quad (A8)$$

$$\lim_{\gamma \to 0} \gamma \langle |Y_i(k, s)|^2 \rangle = \frac{V \times T_i k^2}{4 \pi \omega Z^2 e^2} \Re \langle R_i(k, \omega) \rangle. \quad (A9)$$

These relations also follow, but more readily, as shown by Hagfors, if one additionally assumes that the spatial density fluctuations are independent of the velocities of the particles.

In order to determine the functions $S$ from (18) we consider the vector $E_{ex}$ in cylindrical coordinates, with the axis parallel to $B$. We denote by $z_1$, the angle between $E_{ex}$ and $B$ and by $z_2$ the angle between the component of $E_{ex}$ perpendicular to $B$ and the plane defined by $B$ and $k$. In this coordinate system $E_{ex}$ has the components:

$$E_{ex,x} = E_{ex} \sin z_1 \cos z_2, \quad E_{ex,y} = E_{ex} \sin z_1 \sin z_2, \quad E_{ex,z} = E_{ex} \cos z_1. \quad (A10)$$

This representation of $E_{ex}$ corresponds to that of $v$ in (14). It follows that:

$$E_{ex} \cdot v' = E_{ex} \left[ w \cos q' \sin z_1 \cos z_2 + w \sin q' \sin z_1 \sin z_2 + u \cos z_2 \right]. \quad (A11)$$

9 I. B. Bernstein, Phys. Rev. 109, 10 [1958].
In order to obtain expressions for $S$ similar to that for $R$ we rewrite (A 11):

$$E_{ex}' = E_{ex} \sin \frac{x_1 \cos x_2}{k \sin \psi} (k \omega \sin \psi \cos q' + k \omega \cos \psi) + E_{ex} u \left( \cos \frac{x_1 \cos x_2}{k \sin \psi} \sin \psi \right) + E_{ex} u \sin \psi' \sin \frac{x_1 \sin x_2}{k \sin \psi}. \quad (A 12)$$

Correspondingly we split $S_e(k, s)$ into three terms:

$$S_e(k, s) = S_{e1}(k, s) + S_{eII}(k, s) + S_{eIII}(k, s). \quad (A 13)$$

Since the expression $S_{e1}$ has the same structure as $R$ we have:

$$S_{e1}(k, s) = -\infty \int dx \exp \left[ -s \Omega_e x - \left[ \sin^2 \psi' (1 - \cos x) + \frac{1}{2} x^2 \cos^2 \psi' \right] \Omega_e \frac{k}{m} \right] \sin \frac{x_1 \cos x_2}{k \sin \psi}. \quad (A 14)$$

The second term yields:

$$S_{eII}(k, s) = -i \frac{k \omega}{m} E_{ex}(k, s) \sin \psi \sin \frac{x_1 \cos x_2}{k \sin \psi}. \quad (A 15)$$

and the third term in (A 13) may be reduced to a one-dimensional integral by a procedure similar to that used for the treatment of $R_e$. Then one obtains:

$$S_{eIII}(k, s) = -i \frac{k \omega}{m} E_{ex}(k, s) \sin \psi \sin \frac{x_1 \cos x_2}{k \sin \psi}. \quad (A 16)$$

Similar expressions are valid for $S_i(k, s)$.

(A 13) with (A 14) to (A 16) is the general form of our result. To this point we made no assumptions as to the transvers character of the external electric field, nor was a value assumed for the ratio of its frequency to the electron gyrofrequency.

Let us now make an expansion of $S_e$ for $\omega \gg \Omega_e$. For this purpose we write

$$s = \gamma + i \omega \quad \text{and} \quad \exp(i x) = \cos x + i \sin x$$

and expand the three terms of $S_e$ according to the formulae

$$\int_0^\infty \sin a x f(x) \, dx = \frac{f(0)}{a} - \frac{f''(0)}{a^3} + \frac{f''(0)}{a^5} - \ldots, \quad (A 17)$$

$$\int_0^\infty \cos a x f(x) \, dx = -\frac{f'(0)}{a^2} + \frac{f''(0)}{a^4} - \frac{f''(0)}{a^6} + \ldots \quad (A 18)$$

which are obtained by continued partial integration. The expansions are valid for all functions $f(x)$, for which all derivatives exist and go to zero for $x \to \infty$.

In the special case of the function $S_e$ ($a$ being $\omega/\Omega_e$) the series converge rapidly, if $\omega \gg \Omega_e$. From eqs. (A 14) to (A 16) it follows in the limit $\gamma \to 0$, if all terms of higher order in $B$ are neglected, that

$$S_{e1}(k, s) = -i \frac{k \omega}{m} E_{ex}(k, s) \sin \frac{x_1 \cos x_2}{k \sin \psi}, \quad (A 19)$$

In this case only the component of $E_{ex}$ that is perpendicular to $B$ contributes to $S_e$.

If we finally assume the index of refraction to be unity, i.e. $\omega/k = c$, we have:

$$S_e(k, \omega) = S_{eIII}(k, \omega), \quad (A 23)$$

In the special case that the external electric field is a transvers wave ($E_{ex} k = 0$) the angles $\psi$, $\chi_1$ and $\chi_2$ satisfy the condition

$$\cos \chi_1 \sin \psi = -\sin \chi_1 \cos \chi_2 \sin \psi. \quad (A 22)$$

From this condition follows that with the approximations (A 19) and (A 20) the sum of $S_{e1}$ and $S_{eII}$ vanishes, i.e.: $S_e(k, \omega) = S_{eIII}(k, \omega)$. In this case only the component of $E_{ex}$ that is perpendicular to $B$ contributes to $S_e$.

From eqs. (A 14) to (A 16) it follows in the limit $\gamma \to 0$, if all terms of higher order in $B$ are neglected, that

$$S_{eII}(k, s) = -i \frac{k \omega}{m} E_{ex}(k, s) \sin \frac{x_1 \cos x_2}{k \sin \psi}, \quad (A 20)$$

$$S_{eIII}(k, s) = -\frac{\Omega_e k e E_{ex}(k, s)}{m \omega^2} \sin \frac{x_1 \cos x_2}{k \sin \psi}. \quad (A 21)$$

In the special case of the function $S_e$ ($a$ being $\omega/\Omega_e$) the series converge rapidly, if $\omega \gg \Omega_e$. From eqs. (A 14) to (A 16) it follows in the limit $\gamma \to 0$, if all terms of higher order in $B$ are neglected, that

$$S_{e1}(k, s) = -i \frac{k \omega}{m} E_{ex}(k, s) \sin \frac{x_1 \cos x_2}{k \sin \psi}, \quad (A 19)$$

This is a correction of the result given in ref. 1, which differs from (A 24) by a numerical factor.