Studies on a 3-Terminal Hall Effect Device

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Normal Hall generators are provided with four terminals, two carrying the current and two serving as the so-called Hall electrodes across which the Hall voltage is developed. If these latter terminals are connected externally through a suitable resistance they can jointly be used to serve as a current lead and thus replace one of the current terminals of a normal Hall generator. The device under such condition is essentially a three-terminal one, the terminals forming the vertices of a triangle. It is the purpose of the present note to report the results of some investigations undertaken on such a device.

Experimental Arrangement and Results

A thin rectangular bar of n-type InAs having the dimensions 12×6×.1 mm was chosen. Three small area contacts (1, 3 and 4) as shown in Fig. 1 were made on the sample, care being taken that there was no appreciable contact resistance. Electrical connections were then made as shown in the same figure. Terminals 3 and 4 were properly aligned by adjusting the terminating shunt resistance R so that no voltage was present between them, in the absence of a magnetic field. The device was placed between the pole pieces of an electromagnet and the voltage drops across terminals 1—3, 1—4 and 3—4 (designated as $V_{13}$, $V_{14}$ and $V_{34}$ respectively) were measured for various values of the magnetic field. Measurements were also made by reversing the directions of both current and magnetic field. Results thus obtained are plotted in Figs. 2 and 3 (crosses and hollow circles). The following main features are observed:

(i) With the magnetic field in a particular direction (let us call this direct), the voltage $V_{13}$ is found to increase steadily with $B$ while $V_{14}$ first decreases, passes through a minimum and then begins to rise (Fig. 2, crosses).

(ii) On reversing the magnetic field, the types of variation of the voltages $V_{13}$ and $V_{14}$ are interchanged (Fig. 2, hollow circles). In both cases, reversal of current direction did not produce any change in the results.

(iii) The voltage $V_{34}$ increases almost linearly with $B$ (Fig. 3) having a slope equal to $50.0 \times 10^{-8}$ for both directions of the magnetic field.

Theory of Operation of the Device

The observed characteristics of the device may be explained by considering both the magnetoresistance and Hall effect phenomena in the specimen.

In the absence of any magnetic field, the voltage drops across terminals 1—3 and 1—4 represent the ohmic drops across these terminals. When a magnetic field is applied normal to the direction of current flow,
the voltage across both terminals 1—3 and 1—4 would be affected due to two factors. One of these factors is the well known magnetoresistance effect. Assuming that the magnetoresistance effect of the specimen can be expressed by the relation

$$\frac{\Delta \rho}{\rho_0} = CB^2$$  \hspace{1cm} (1)

where $\Delta \rho$ is the change in resistivity due to application of a magnetic field $B$, $\rho_0$ is resistivity in absence of any magnetic field, and $C$ and $x$ are constants for the material of the specimen under investigation, it follows that the voltage drop across terminals 1—3 and 1—4 would each increase by an amount $V_0 CB^2$ where $V_0$ is the voltage drop in zero magnetic field.

The other factor responsible for the change of $V_{13}$ and $V_{14}$ is the Hall effect. While an exact expression for the magnitude of this change can not be readily given, one may as a first approximation assume this to be proportional to $B$ and also to the current in the other branch. Further, if $V_{13}$ rises due to this effect, $V_{14}$ would fall and vice versa. Thus the expressions for $V_{13}$ and $V_{14}$ as function of magnetic field $B$ may be written as

$$V_{13} = V_0 + bi_{14}B + V_0 CB^x$$ \hspace{1cm} (2)

$$V_{14} = V_0 - bi_{13}B + V_0 CB^x$$ \hspace{1cm} (3)

where $i_{13}$ and $i_{14}$ are currents in the branches 1—3 and 1—4 respectively and $b$ is a constant of proportionality. The nature of Eq. (3) suggests that $V_{14}$ would pass through a minimum as $B$ is increased (since $x$ is usually greater than 1). To obtain the position of the minimum, differentiating Eq. (3) and equating to zero give

$$B_{\text{min}} = \left( \frac{i_{13}b}{xV_0C} \right)^{1/(x-1)}.$$ \hspace{1cm} (4)

As $B$ is increased beyond $B_{\text{min}}$, $V_{14}$ begins to rise and would attain the value $V_0$ at some value of the magnetic field $B$. Designating the latter by $B_0$, say, we have

$$B_0 = \left[ \frac{i_{13}b}{V_0C} \right]^{1/(x-1)}.$$ \hspace{1cm} (5)

To obtain an expression for $V_{34}$ we note from Eqs. (2) and (3), that

$$V_{34} = V_{13} - V_{14} = (i_{13} + i_{14}) bB.$$ \hspace{1cm} (6)

Eq. (6) enables us to find out the constant $b$ from the slope of the $V_{34}$ versus $B$, provided the total current through the sample is known.

Again, by utilising Eqs. (4) and (5) one obtains

$$x = \left[ b_0/B_{\text{min}} \right]^{x-1} \quad \text{or} \quad x^{1/(x-1)} = \left[ b_0/B_{\text{min}} \right].$$ \hspace{1cm} (7)

Hence $x$ can be calculated. Further, putting the value of $x$ into Eqs. (4) or (5), $C$ can be obtained.

Testing the Validity of the Theory

In order to check the validity of the theory outlined above, the experimental results reported were compared with those predicted by the theory.

Let us begin the discussion with Eq. (6) which shows that the plot of $V_{34}$ versus $B$ should be a straight line. As shown in Fig. 3 this characteristic is indeed almost linear. Again, from the slope of this characteristic, since the total current through the sample is known (10 mA) one can determine $b$ which comes out to be $5.0 \times 10^{-8}$ cm$^2$/coulomb. Now, the Hall coefficient $R_{\text{H}}$ of the sample was found from Hall effect measurements to be $66$ cm$^2$/coulomb. Dividing this by the thickness $t$ of the sample one obtains a value of $6.6 \times 10^{-8}$ for $R_{\text{H}}/t \times 10^{-8}$ which is in fair agreement with the value of $b$ as reported above. This confirms the assumption made in the preceding section, viz., that the voltage $V_{34}$ is due to the well known Hall effect.

Now, as mentioned in connection with Eq. (7) both $x$ and $C$ can be determined by measuring the values of $B_0$ and $B_{\text{min}}$. The values of $x$ and $C$ thus obtained are 1.8 and 3.62 $\times 10^{-8}$ (Gauss)$^{-1}$ respectively. Utilizing these values of $C$ and $x$ and knowing the values of $V_0$, $b$, $i_{13}$ and $i_{14}$ the voltages $V_{13}$ and $V_{14}$ were calculated theoretically for different values of $B$, using Eqs. (2) and (3), and are plotted in Fig. 2. It is seen that the theoretical values are in satisfactory agreement with the experimental results. This proves the validity of Eqs. (2) and (3).

Concluding Remarks

The three terminal device can be used in almost all applications where the conventional four terminal Hall generators are used. Other applications of the device could also be found utilizing the simultaneous application of the Hall and magnetoresistance effects, as, for example, the development of an analogue function generator. Investigations along this line are in progress.

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